GAUGE INVARIANCE FOR THE MASSIVE AXION

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ABSTRACT

A massive gauge invariant formulation for scalar (ϕ) and antisymmetric (C_{mnp}) fields with a topological coupling, which provides a mass for the axion field, is considered. The dual and local equivalence with the non-gauge invariant proposal is established, but on manifolds with non-trivial topological structure both formulations are not globally equivalent.

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1 INTRODUCTION

In four dimensions a massless (pseudo)scalar field: the axion, is dual to the antisymmetric field B_{mn} (only if derivative couplings are considered, therefore massive terms are excluded) as a particular case of the general duality between p and $D-p-2$ forms in D dimensions. Since non-perturbative effects break the local Peccei-Quinn symmetry for the axion field, in order to give mass to the axion, the duality between a massive axion and an antisymmetric field was considered an enigma until two independent approaches [1], [2] were developed recently. Now, is understood that take into account non-perturbative effects, the usual duality between a massless scalar field and an antisymmetric field B_{mn} is not broken, but replaced by the duality between a massive scalar ϕ and an massive antisymmetric field C_{mnp}. An early attempt to undestand this duality was considered in the reference[3] A characteristic feature of this duality is the lost of abelian gauge invariance for the antisymmetric field. In this article, we will show that a gauge invariant theory, which involved a topological coupling and considered several years ago [4] in the context of the $U(1)$ problem, is locally equivalent to the non gauge invariant proposal. This equivalence is similar to what happen in three dimensions for massive topologically and self-dual theories [5] and the Proca and massive topologically gauge invariant theories in four dimensions [6]. We will study the equivalence through the existence of a master action from which local and global considerations are established.

2 THE GAUGE INVARIANT MODEL

An illustrative model for the massive axion is given by the following master action[7]

$$I =< -\frac{1}{2}v_m v^m + \phi\partial_m v^m + \frac{1}{2}m^2\phi^2 >, \tag{1}$$

where v_m is a vector field and ϕ is a scalar field($<>$ denotes integration in four dimensions). Eliminating the field v_m through its equation of motion: $v_m = \partial_m\phi$, the action for a massive scalar is obtained, while using the equation of motion, obtained by varying the scalar field ϕ ($\phi = \frac{1}{m^2}\partial_m v^m$), we have

$$I_v = \frac{1}{2} < v_m v^m + \frac{1}{m^2}(\partial_m v^m)^2 > \tag{2}$$

and the propagator corresponding to the field v_m is $\eta_{mn} - \frac{k_m k_n}{k^2+m^2}$, which is just equal to those discussed in [1]. A simple way to show the duality, rely on introducing the dual of the vector field $v^m = \frac{1}{3!} m \epsilon^{mnpq} C_{npq}$ in the action I_v, yielding the master action

$$I_{M1} =< -\frac{1}{2.3!} m^2 C^{mnp} C_{mnp} + \frac{1}{3!} m \epsilon^{mnpq} \phi \partial_m C_{npq} - \frac{1}{2} m^2 \phi^2 > . \tag{3}$$

from which the duality is easily infered. In fact, eliminating the scalar field C_{mnp} (or ϕ) through its equation of motion, the action for a massive scalar field (or the massive antisymmetric field C_{mnp}) is obtained. In anycase, the gauge invariance is spoiled. Now, we can ask whether there really exist an invariant gauge theory compatible with a massive term for the axion field. The answer is possitive. We will show that the following action

$$I_{M2} =< -\frac{1}{2} \partial_m \phi \partial^m \phi - \frac{1}{2.4!} G_{mnpq} G^{mnpq} - \frac{m}{6} \epsilon^{mnpq} C_{mnp} \partial_q \phi >, \tag{4}$$

where $G_{mnpq} \equiv \partial_m C_{npq} - \partial_n C_{mpq} + \partial_p C_{mnq} - \partial_q C_{mnp}$ is the field strenght associated to the antisymmetric field C_{mnp}, is locally equivalent to I_{M1}, describing the propagation of a massive scalar excitation: a massive axion. Note that the coupling term is an extension of the usual BF term and the action is invariant under the abelian gauge transformations

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi \phi = 0. \tag{5}$$

This action was considered previously in ref [4]. as a generalization to four dimensions of the Schwinger model in two dimensions.

Let us see, how this action is related to the propagation of a massive axion and why the equivalence with the non gauge invariant action must hold. Rewritten down the action (eq. (4)) by introducing $F^{mnp} \equiv \epsilon^{mnpq} F_q$ as the dual tensor of $F_m = \partial_m \phi$, we can eliminate F^{mnp} through its equation of motion: $F^{mnp} = -mC^{mnp}$ and substituing, the action for the massive antisymmetric field C^{mnp} appears. Going on an additional step, the dual of the antisymmetric field $C^{mnp} = \frac{1}{m} \epsilon^{mnpq} v_q$ is introduced, and the action for the vector field v_m, eq. (2), is obtained. On the other hand, if we introduce $\lambda \equiv -\frac{1}{4} \epsilon^{mnpq} G_{mnpq}$ as the dual of the strenght field G_{mnpq} into the action (4), we observe that λ plays the role of an auxiliary field, whose elimination through its equation of motion ($\lambda = -m\phi$) lead to the action of a massive scalar field.

It is worth recalling, since the action is expressed only in derivatives of the scalar field, that the dual theory can be achieved, reemplacing $\partial_m\phi$ by $\frac{1}{2}l_m$ and add a BF term: $\frac{1}{4}l_m\epsilon^{mnpq}\partial_n B_{pq}$[8]. The dual action is[4]

$$I_d = < -\frac{1}{2.4!}G_{mnpq}G^{mnpq} - \frac{1}{2.3!}(mC_{mnp} - H_{mnp})(mC^{mnp} - H^{mnp}) >, \quad (6)$$

where $H_{mnp} = \partial_m B_{np} + \partial_n B_{pm} + \partial_p B_{mn}$ is the field strength of the antisymmetric field B_{mn}, which was introduced in the BF term. This action just describes the interaction of open menbranes whose boundaries are closed strings [9] and is invariant under the following gauge transformations

$$\delta C_{mnp} = \partial_m\xi_{np} + \partial_n\xi_{pm} + \partial_p\xi_{mn}, \quad \delta B_{mn} = \partial_m\lambda_n - \partial_n\lambda_m - m\xi_{mn}. \quad (7)$$

The ξ gauge transformation allows us gauged away the antisymmetric field B_{mn}, leading to the massive antisymmetric field C_{mnp} action.

3 THE EQUIVALENCE

Now, we are going on to show the equivalence. Let us take the following master action

$$I_M = < -\frac{1}{3!}m^2 a_{mnp}a^{mnp} - \frac{1}{2!}m^2\psi^2 + \frac{1}{4!}m\epsilon^{mnpq}\psi G_{mnpq} + \frac{1}{3!}m\epsilon^{mnpq}(a_{mnp} - C_{mnp})\partial_q\phi > . \quad (8)$$

Independent variations in a_{mnp}, ψ, C_{mnp} and ϕ lead to the following equations of motion

$$a^{mnp} = \frac{1}{m}\epsilon^{mnpq}\partial_q\phi, \quad (9)$$

$$\psi = \frac{1}{4!m}\epsilon^{mnpq}G_{mnpq}, \quad (10)$$

$$\epsilon^{mnpq}\partial_m(\psi - \phi) = 0, \quad (11)$$

and

$$\epsilon^{mnpq}\partial_q(a_{mnp} - C_{mnp}) = 0. \quad (12)$$

Replacing the expressions for a_{mnp} and ψ given by eqs. (9) and (10) into I_M, the gauge invariant action I_{M2} is obtained. On the other hand, the solutions of the equations of motion (11) and (12) are

$$\phi - \psi = \omega, \quad C_{mnp} - a_{mnp} = \Omega_{mnp}, \quad (13)$$

where ω and Ω_{mnp} are 0 and 3-closed forms, respectively. Locally, we can set

$$\omega = constant, \quad \Omega_{mnp} \equiv L_{mnp} = \partial_m l_{np} + \partial_n l_{pm} + \partial_p l_{mn}, \tag{14}$$

and subtituing into I_M, we obtain the following "Stuckelberg" action

$$I_s =< -\frac{1}{3!}m^2(C_{mnp}-L_{mnp})(C^{mnp}-L^{mnp})-\frac{1}{2}m^2(\phi-\omega)^2+\frac{1}{4!}m\epsilon^{mnpq}(\phi-\omega)G_{mnpq} > . \tag{15}$$

This action is invariant under

$$\delta_\xi C_{mnp} = \partial_m \xi_{np} + \partial_n \xi_{pm} + \partial_p \xi_{mn}, \quad \delta_\xi l_{mn} = \xi_{mn}, \tag{16}$$

which allow us gauged away the l_{mn} field and recover I_{M1} (we have redefined $\phi - \omega$ as ϕ since ω is a constant). In this way, the local equivalence is stated. This local equivalence can also be established from a hamiltonian point of view and will be reported elsewhere [10]. In an ample sense, we must consider $\psi = \phi - \omega$ and $a_{mnp} = C_{mnp} - \Omega_{mnp}$ as the general solutions and I_{M2} is locally and globaly equivalent to

$$\begin{aligned} \bar{I}_{M1} &= < -\frac{1}{3!}m^2(C_{mnp} - \Omega_{mnp})(C^{mnp} - \Omega^{mnp}) - \frac{1}{2!}m^2(\phi - \omega)^2 \\ &+ \frac{1}{4!}m\epsilon^{mnpq}(\phi - \omega)G_{mnpq} - \frac{1}{3!}m\epsilon^{mnpq}\Omega_{mnp}\partial_q\phi >, \end{aligned} \tag{17}$$

which is an adequate extension of I_{M1}.

Finally, we can eliminate ϕ and C_{mnp} to achieve

$$\bar{I}_{M1} = I_{M1[a,\psi]} - I_{top[\omega,\Omega]}, \tag{18}$$

where

$$I_{top[\omega,\Omega]} =< \frac{1}{3!}m\epsilon^{mnpq}\Omega_{mnp}\partial_q\omega > \tag{19}$$

is the extension of the BF term for the topological coupling between 0 and 3-forms in four dimensions. From this result, we have that the partition funtions of I_{M1} and I_{M2} differ by a topological factor.

$$Z_{M2} = Z_{top} Z_{M1} \tag{20}$$

In general, on manifolds with non trivial topological structure $Z_{top} \neq 1$. Only when the manifold has a trivial structure, we must have $Z_{top} \equiv 1$, reflecting the local equivalence.

Sumarizing, we have seen that a gauge invariant description for massive axions is possible which is (locally)equivalent to the non-gauge invariant proposal. Several aspects of this proposal are under considerations: a detailed hamiltonian description for both proposal of generating mass for the axion and a complete BRST analysis of the gauge invariant model considered in this paper[10].

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