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SO(10) Theory of R-parity and Neutrino Mass

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Abstract

We study the Higgs sector of a SO(10) grand unified theory which predicts exact conservation of R-parity at all scales and incorporates the see-saw mechanism. We find possible intermediate scales and light states compatible with the constraints coming from the running of the gauge couplings. Such a pattern could lower the SO(10) breaking scale, allowing the $d = 6$ proton decay operators to be comparable in magnitude to the $d = 5$ ones.

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I. INTRODUCTION

It is well-known that SO(10) grand unified theory offers an appealing framework for the unification of quarks and leptons and their forces. It is the minimal theory which unifies a family of fermions in a simple irreducible spinorial representation. The left-right (LR) symmetry in the form of charge conjugation [1] is a finite gauge transformation and thus automatically built into the theory. Last but not least, it naturally incorporates the see-saw mechanism [2] which provides a rationale for the smallness of the neutrino mass.

However, in its non-supersymmetric realization it fails to provide a "canonical" see-saw formula [3]. It is here that supersymmetry (for a recent review on supersymmetric grand unified theories see for example [4]) plays an important role and in certain cases the see-saw can take the canonical form. We study this issue in the context of the SO(10) theory and find, unfortunately, a situation similar to the ordinary case. More important, the see-saw mechanism actually gives us the low energy effective theory of supersymmetry. What happens is that R-parity remains an exact symmetry at all scales. This is a fundamental result which guarantees the stability of the lightest supersymmetric partner (LSP), an ideal candidate for the dark matter of the universe. This is true of any renormalizable theory of the see-saw mechanism based on the spontaneous breaking of B$-$L symmetry [5].

Without unification one can not predict the value of the right-handed neutrino mass scale. In this sense SO(10) is an ideal theory especially in its supersymmetric version. It incorporates all the above features and it helps narrow down the range in which the right-handed neutrino mass lies.

In this paper we construct a complete SO(10) theory of see-saw mechanism and R-parity. After a careful study of symmetry-breaking patterns, we compute the particle spectra and perform the unification analysis. We find a plethora of states (often carrying color or a large electromagnetic charge) whose masses, due to supersymmetry, could in principle lie much below the associated symmetry breaking scales. The reason for this is the violation of the survival principle, which we discussed at length in [6]. There we have coined the term "survival of the fittest" for this phenomenon. It is due to the absence of some quartic couplings in the potential (i.e., the absence of some trilinear couplings in the superpotential), as is often the case in supersymmetric models.

The rest of this paper is organized as follows: in the next section we discuss the salient features of the SO(10) theory with a renormalizable see-saw and the possible symmetry breaking patterns. In Section III we give detailed analysis of the particle spectrum and in Section IV we present the unification constraints. In Section V the phenomenological and cosmological consequences are discussed at some lengths. Section VI is devoted to the summary. The technical and computational details of symmetry breaking are left for the Appendices.

II. THE SO(10) SUPERSYMMETRIC THEORY WITH THE RENORMALIZABLE SEE-SAW MECHANISM.

Supersymmetric SO(10) models have been studied at length [9], but almost exclusively with the non-renormalizable version of the see-saw mechanism. More precisely, one chooses one (or more) pair of Higgs fields in the spinorial representation **16** and $\overline{\mathbf{16}}$ whose VEVs induce B-L breaking and the mass for the right-handed neutrino through the $d = 4$ terms: $m_{\nu_R} \simeq \langle \mathbf{16}\overline{\mathbf{16}} \rangle / M_{Pl}$. The disadvantage of this program is that then R-parity is broken at a large scale M_R, and thus one needs additional, often ad-hoc symmetries to understand the smallness of R-parity breaking at low energies.

Our motivation is orthogonal to this. We wish to have a theory of R-parity and this points immediately to the renormalizable version of the see-saw mechanism. To see this, recall first that under R-parity $p \to p$, $\tilde{p} \to -\tilde{p}$ (where p stands for particles and $\tilde{p}$ for sparticles of the supersymmetric standard model) can be written as (S is the spin)

$$R = (-1)^{3(B-L)+2S} \equiv M(-1)^{2S} \,, \tag{1}$$

where the so-called M parity is obviously equivalent to R (the factor $(-1)^{2S}$ becomes 1 for the physical Hamiltonians and only scalars with $S = 0$ are allowed to have non-vanishing VEVs).

One of the most appealing aspects of the supersymmetric SO(10) is that M is a finite gauge symmetry, since under M

$$\mathbf{16} \xrightarrow{M} -\mathbf{16} \quad , \quad \mathbf{10} \xrightarrow{M} \mathbf{10} \,, \tag{2}$$

and all other representations built out of the fundamental **10**, such as **45**, **54**, **126**, etc. are even. The symmetry in (2) is simply C^2, where C is the center of SO(10), so that under it **16**$\rightarrow i$ **16**, **10**$\rightarrow$ **−10**. This points strongly towards using a 126-dimensional Higgs for the breaking of B−L and the see-saw mechanism (for previous analysis of SO(10) models with **126** representations see [10–12]). One drawback of these representations is their huge contribution to the SO(10) β function, so that the Landau pole is reached very soon above M_X. We will be interested only in the physics below or at the unification scale, where the couplings are perturbative.

We wish to construct a renormalizable SO(10) theory with a see-saw, and this requires the minimum set of Higgs representations which break SO(10) down to the MSSM:

$$S = \mathbf{54}\ , \quad A = \mathbf{45}\ , \quad \Sigma = \mathbf{126}\ , \quad \overline{\Sigma} = \overline{\mathbf{126}}\ . \tag{3}$$

Although SO(10) is anomaly-free, as is well-known, one has to use both Σ and $\overline{\Sigma}$ in order to ensure the flatness of the D-piece of the potential at large scales $\gg M_W$.

What are the possible channels of symmetry breaking and why do we need all the above fields? Of course, one possible channel strongly encouraged by the MSSM unification constraints is through SU(5). Another possibility, much less studied, is the intermediate Pati-Salam (PS) and/or LR scale. For this purpose, it is useful to know the decomposition of the above fields under the PS gauge group $SU(2)_L\times SU(2)_R\times SU(4)_C$

$$\begin{aligned}
S\ &= \mathbf{54} = (1,1,1) + (1,1,20) + (3,3,1) + (2,2,6)\ ,\\
A\ &= \mathbf{45} = (1,1,15) + (3,1,1) + (1,3,1) + (2,2,6)\ ,\\
\Sigma &= \mathbf{126} = (3,1,\overline{10}) + (1,3,10) + (2,2,15) + (1,1,6)\ ,\\
\overline{\Sigma} &= \overline{\mathbf{126}} = (3,1,10) + (1,3,\overline{10}) + (2,2,15) + (1,1,6)\ .
\end{aligned} \tag{4}$$

One may think that it is redundant to take both A and S together with Σ and $\overline{\Sigma}$. After all either $(1,1,1)$ or $(1,1,15)$ could produce the first stage of symmetry breaking, down to the PS or LR symmetry, to be followed by $(1,3,10)$ in Σ. Notice that the $(1,3,10)$ field contains a color singlet component Δ_c which couples to the right-handed neutrino ν^c in **16**, and through $\langle\Delta_c\rangle \equiv M_R$ produces the see-saw mechanism. Since ν^c is an SU(5) singlet, so is obviously $\langle\Delta_c\rangle$. Thus the possible $\langle\mathbf{126}\rangle \equiv \langle\Delta_c\rangle \neq 0$ can break SO(10) only down to SU(5).

Clearly, either $\langle A\rangle \neq 0 \neq \langle\Sigma\rangle$ or $\langle S\rangle \neq 0 \neq \langle\Sigma\rangle$ suffices to break the SO(10) symmetry all the way down to the MSSM. So why is it that we need both A and S fields in the renormalizable version of the theory which we study here ?

To see this, we need to write down the most general superpotential containing S, A, Σ and $\overline{\Sigma}$

$$\begin{aligned}
W = &\frac{m_S}{2}\mathrm{Tr}\, S^2 + \frac{\lambda_S}{3}\mathrm{Tr}\, S^3 + \frac{m_A}{2}\mathrm{Tr}\, A^2 + \lambda \mathrm{Tr}\, A^2 S\\
&+ m_\Sigma \Sigma\overline{\Sigma} + \eta_S \Sigma^2 S + \overline{\eta}_S \overline{\Sigma}^2 S + \eta_A \Sigma\overline{\Sigma} A\ .
\end{aligned} \tag{5}$$

The above form is only symbolic, details are given in Appendix A.

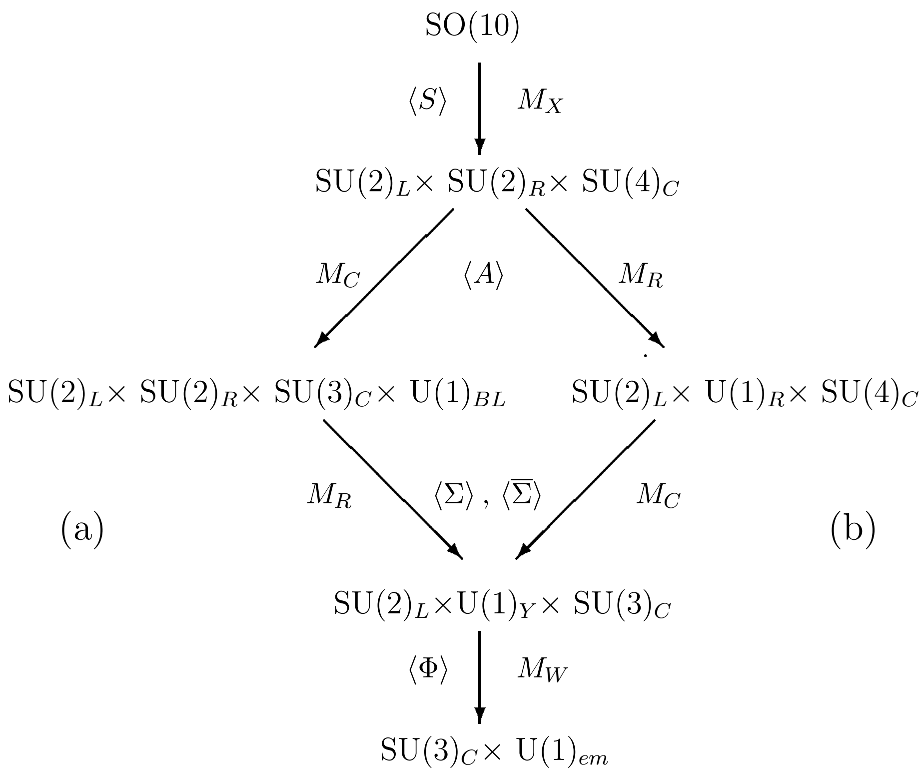
It is trivial to see why A, Σ and $\overline{\Sigma}$ cannot suffice. From

$$F_A = m_A A + \eta_A \Sigma\overline{\Sigma} \tag{6}$$

and the fact that $\langle\Sigma\overline{\Sigma}\rangle$ preserves SU(5), it is clear that $\langle A\rangle$ must be SU(5) invariant, too. In order to achieve more breaking, one may resort to non-renormalizable operators of the type $\mathrm{Tr}\, A^4/M$, $\mathrm{Tr}\,(\Sigma\overline{\Sigma})^2/M$, etc. (where M is some new large scale). We prefer to include another field S; after all if $\langle S\rangle \gg \langle A\rangle$, we will get the non-renormalizable terms after integrating out the heavy fields.

It is also straightforward to see why S does not suffice together with Σ and $\overline{\Sigma}$. The point is that the η_S and $\overline{\eta}_S$ terms cannot produce any interaction between the VEV acquiring fields $(1,1,1)_S$ and $(1,3,10)_\Sigma$: there is no singlet in the tensor product 10×10 in SU(4); the same argument applies for $\overline{\Sigma}$. Thus, although $\langle S\rangle$ can break SO(10) down to PS, the lack of interactions for the $(1,3,10)$ and $(1,\overline{3},10)$ fields forces their VEVs to vanish.

In short, in the renormalizable theory one needs all the above fields. One can envision the following physically interesting patterns of symmetry breaking, which we label **(a)** and **(b)**



(7)

It turns out that both of these chains are quite interesting in the sense that they lead to a plethora of "light" states, i.e., states whose masses lie below the corresponding symmetry breaking scale.

In order to study any of the chains, we need the conditions for the F and D flatness at the scales $\gg M_W$. This again is discussed at length in Appendix A, here we address only the salient features. The F-flatness equations are

$$F_{(1,1,1)_S} = m_s s + \frac{1}{2}\lambda_S s^2 + \frac{2}{5}\lambda(a^2 - b^2) = 0\ ,$$

$$F_{(1,1,15)_A} = m_A a + 2\lambda a s + \frac{1}{2}\eta_A \sigma\bar{\sigma} = 0\ ,$$

$$F_{(1,3,1)_A} = m_A b - 3\lambda b s + \frac{1}{2}\eta_A \sigma\bar{\sigma} = 0 \ ,$$

$$F_{(1,3,10)_\Sigma} = \sigma \left[m_\Sigma + \eta_A(3a+2b) \right] = 0 \ ,$$

$$F_{(1,3,10)_{\overline{\Sigma}}} = \bar{\sigma} \left[m_\Sigma + \eta_A(3a+2b) \right] = 0 \ , \tag{8}$$

where

$$s = \langle (1,1,1)_S \rangle \quad , \quad a = \langle (1,1,15)_A \rangle \quad , \quad b = \langle (1,3,1)_A \rangle \quad ,$$
$$\sigma = \langle (1,3,10)_\Sigma \rangle \quad , \quad \bar{\sigma} = \langle (1,3,\overline{10})_{\overline{\Sigma}} \rangle \quad . \tag{9}$$

All other fields have zero vacuum expectation values (see Appendix A). Notice that the choice of the two chains of symmetry breaking depends on the ratio of a and b. In both cases it is assured that s is the largest VEV.

Next, we can imagine two possibilities:

(a) $s \gg a \gg \sigma = \bar{\sigma} \gg b \simeq \sigma^2/s$,

(b) $s \gg b \gg \sigma = \bar{\sigma} \gg a \simeq \sigma^2/s$,

which correspond precisely to chains (a) and (b) respectively (notice that $\sigma = \bar{\sigma}$ is necessary for D-flatness, see Appendix A). This can be achieved by paying the usual price of fine-tuning, an issue which is beyond the scope of this paper. In case (a) this implies

$$m_A + 2\lambda s \simeq \frac{\sigma^2}{a} \ll s \ , \tag{10}$$

which then ensures

$$b \simeq \frac{\sigma^2}{s} \ll \sigma \tag{11}$$

(it is important to keep in mind that b can never vanish). In case (b), of course, the conditions are obtained interchanging the roles of a and b, i.e. $m_A - 3\lambda s \simeq \sigma^2/b \ll s$.

A comment is in order. In the F flatness conditions (8) we ignore the fields in **16** and **10** dimensional representations. This is justifiable for the Standard Model non-singlet fields, but not for $\tilde{\nu^c}$ in **16**. However, as we already noticed, $\tilde{\nu^c}$ is coupled to the SU(5) singlet in $\overline{\Sigma}$ which gets a VEV $\bar{\sigma}$. The F-equations will then give

$$F_{\tilde{\nu^c}} = \sigma \langle \tilde{\nu^c} \rangle = 0 \tag{12}$$

guaranteeing a vanishing VEV for $\tilde{\nu^c}$. This is a general feature of theories with a renormalizable see-saw mechanism, as shown in [5]. We readdress this important issue below, when we discuss the fate of R-parity.

A few words about the low-energy sector of the theory. Besides the above fields we also need the usual 16-dimensional representations, which give the standard model fermions and sfermions and representations containing the electroweak Higgs doublets. For the sake of minimality we choose for the latter the 10-dimensional representation. We leave their number open in order to be as general as possible. It is well known that the minimal theory with just one **10** predicts wrong fermion masses and no CKM mixing if one restricts oneself to the tree level and/or renormalizable interactions. The higher dimensional operators can easily introduce small flavor mixings and correct the light quark mass ratios. Recently it has been emphasized that the radiative corrections in supersymmetric models can also do the job [13]. We present our results for both the minimal model and the model with two 10-dimensional Higgs supermultiplets, since the latter certainly works.

III. PARTICLE SPECTRUM.

Since we wish to determine the scales of symmetry breaking, we need the precise particle spectrum of both light ($\sim M_W$) and heavy ($\gg M_W$) states. The point here is that in supersymmetric theories one can not rely on the survival (or extended) principle, since the lack of cubic terms in the superpotential for many representations naturally suppresses many particle masses [6]. More precisely, if there is an intermediate scale M_I besides the unification scale M_X, then one finds the effective quartic couplings suppressed by M_I/M_X and there are a number of states whose masses become of the order of M_I^2/M_X instead of M_I. This of course has a strong impact on the unification predictions. This will be made manifest in the examples discussed below.

Case (a)

It is a simple exercise to show the well-known fact that all the states in **54** become super heavy. So do most of the states in **45**, **126** and $\overline{\mathbf{126}}$. The only states which do not pick up a mass of order $s \simeq M_X$ in **126** and $\overline{\mathbf{126}}$ are the SU(2) triplets, namely $(3,1,\overline{10})_\Sigma$, $(1,3,10)_\Sigma$, $(3,1,10)_{\overline{\Sigma}}$, and $(1,3,\overline{10})_{\overline{\Sigma}}$. As mentioned above we must fine-tune the mass of the field $(1,1,15)_A$ $(m_A + 2\lambda s \simeq 0)$.

Next, we switch-on $a \simeq M_C$ and $\sigma \simeq M_R$ with $a \gg \sigma$. As is clear from (7), $a \simeq M_C$ breaks PS down to LR; it is the color singlet component in $(1,1,15)_A$ that gets a VEV. Due to the supersymmetric version of the Higgs mechanism, color triplets in $(1,1,15)_A$ get a mass of order a, but the color octet remains in principle much lighter. Due to the absence of the trilinear term A^3, it can only get a mass of order M_C^2/M_X (through the mixing with the color octet in $(1,1,20)_S$) or M_R^2/M_C (through the fine-tuning condition $m_A + 2\lambda s \simeq \sigma^2/a$).

Regarding the fields in Σ and $\overline{\Sigma}$, notice first that the neutral component in $(1,1,15)_A$ is a parity (charge conjugation) odd super-Higgs. In other words, the left-handed triplets $(3,1,\overline{10})_\Sigma$ and $(3,1,10)_{\overline{\Sigma}}$ get masses $m_\Sigma - 3a\eta_A \simeq M_C$ and decouple for lower energies. Similarly, the color triplet and sextet fields in their right-handed counterparts $(1,3,10)_\Sigma$ and $(1,3,\overline{10})_{\overline{\Sigma}}$ get masses $\sim M_C$.

Except for the doubly-charges states δ_c^{++} and $\bar{\delta}_c^{++}$, the rest of the fields in the color singlet components for the above representations get masses $\sim M_R$ through the super-Higgs mechanism. The δ_c^{++} and $\bar{\delta}_c^{++}$ fields, much as in LR supersymmetric models [14], pick up their mass only through $b \simeq \sigma^2/s$.

The masses are summarized in Table 1 below.

State	Mass
all of S in **54** all of A in **45**, except $(1,1,15)_A$ all of Σ in **126** + $\overline{\Sigma}$ in $\overline{\mathbf{126}}$, except $SU(4)_C$ decuplets	$\sim M_X$
$(3,1,\overline{10})_\Sigma + (3,1,10)_{\overline{\Sigma}}$ color triplets and sextets of $(1,3,10)_\Sigma$ and $(1,3,\overline{10})_{\overline{\Sigma}}$ color triplets of $(1,1,15)_A$	$\sim M_C$
$(\delta_c^0 - \overline{\delta}_c^0), \delta_c^+, \overline{\delta}_c^+$ from the color singlets of $(1,3,10)_\Sigma$ and $(1,3,\overline{10})_{\overline{\Sigma}}$	$\sim M_R$
color octet and singlet of $(1,1,15)_A$	$\sim M_1 \equiv Max\left[\frac{M_R^2}{M_C}, \frac{M_C^2}{M_X}\right]$
$(\delta_c^0 + \bar{\delta}_c^0), \delta_c^{++}, \bar{\delta}_c^{++}$ from the color singlets of $(1,3,10)_\Sigma$ and $(1,3,\overline{10})_{\overline{\Sigma}}$	$\sim M_2 \equiv M_R^2/M_X$

Table 1: Mass spectrum for the symmetry breaking chain $SO(10)\stackrel{M_X}{\rightarrow}SU(2)_L\times SU(2)_R\times SU(4)_C\stackrel{M_C}{\rightarrow} SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_C \stackrel{M_R}{\rightarrow} SU(2)_L\times U(1)_Y\times SU(3)_C$.

Notice the 'light' states in the two last rows of the type we discussed above as the violation of the survival principle. Their small masses are the product of the lack of renormalizable interactions and fine-tuning conditions.

On top of this there are of course three generations of fermions and sfermions which lie at M_W or below. The right-handed neutrino supermultiplet is at M_R, but it does not affect the running. In the minimal model the 10-dimensional Higgs multiplet splits into a light ($\sim M_W$ or supersymmetry breaking scale M_S) bidoublet, which provides the two MSSM doublets, and the superheavy ($\sim M_X$) color triplet and antitriplet. If there are two **10**s, then besides the two light doublets and all the superheavy colored states we will have two more doublet superfields with masses $>> M_W$. The range for the heavy doublet masses depend on one's scenario for generation of the weak mixing angles. If these are to arise at the tree level from bi-doublet mixing induced by the vev of the (1,3,1) submultiplet of the **45**, then one needs more than one 10-plet and it is necessary to assume $M_H \sim b$ (where M_H is the coefficient of the $\mathbf{10}^2$ term in the superpotential), while the coupling $\lambda_H \sim b/M_X$ (where λ_H is the coefficient of the $\mathbf{54}\ \mathbf{10}^2$ term in the superpotential), in which case the extra doublets have mass $\sim M_R^2/M_X$. Else the mixing is negligible and the doublets are superheavy so that the theory effectively reduces to the case with a single **10**. One may obtain realistic mixing even with a single 10-plet if radiative corrections due to soft terms are appropriate [13].

Case (b) Let us now turn to case (b) in (7). This implies interchanging the values of the **45** VEVs a and b. The analysis proceeds along the same lines, so we just present the particle spectrum in Table 2.

State	Mass
all of S in **54** all of A in **45**, except $(3,1,1)_A + (1,3,1)_A$ all of Σ in **126** + $\overline{\Sigma}$ in $\overline{\mathbf{126}}$, except $SU(4)_C$ decuplets	$\sim M_X$
$(3,1,\overline{10})_\Sigma + (3,1,10)_{\overline{\Sigma}}$ $(1, {}^{0}_{-}, 10)_\Sigma$ and $(1, {}^{+}_{0}, \overline{10})_{\overline{\Sigma}}$ $\omega_c^\pm$ from $(1,3,1)_A$	$\sim M_R$
color triplets and singlets from $(1,+,10)_\Sigma$ and $(1,-,\overline{10})_{\overline{\Sigma}}$	$\sim M_C$
$(3,1,1)_A$	$\sim M_1 \equiv Max\left[\frac{M_R^2}{M_X}, \frac{M_C^2}{M_R}\right]$
color sextets from $(1,+,10)_\Sigma$ and $(1,-,\overline{10})_{\overline{\Sigma}}$	$\sim M_2 \equiv M_C^2/M_X$

Table 2: Mass spectrum for the symmetry breaking chain $\mathrm{SO}(10) \xrightarrow{M_X} \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(4)_C \xrightarrow{M_R} \mathrm{SU}(2)_L \times \mathrm{U}(1)_R \times \mathrm{SU}(4)_C \xrightarrow{M_C} \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_C$. The states in $(1,3,10)$ and $(1,3,\overline{10})$ were decomposed according to their T_{3R} number, for example $(1,+,10)$ denotes the component of $(1,3,10)$ with $T_{3R} = +1$, etc.

Notice again the survival of the fittest principle. These are the color sextets and left-handed triplet states. Again their small masses are due to the lack of renormalizable interactions or the fine-tuning condition ($m_A - 3\lambda s \simeq \sigma^2/b \simeq M_C^2/M_R$).

In this case the heavy bi-doublets will have masses of order $b \sim M_R$ provided M_H, λ_H are again chosen small. Else one may again use a single 10-plet and radiative effects.

IV. UNIFICATION OF GAUGE COUPLINGS

Armed with the complete spectrum of the physical states we are now ready to perform the analysis of the unification constraints. Notice that this is a precise analysis without any *ad hoc* assumptions made regarding the spectrum at intermediate scales. At this stage we wish to have a rough qualitative estimate of the new mass scales and thus it is only appropriate to perform this at the one-loop level. In this way we check whether the assumptions (a) and (b) are consistent with the low-energy values of the coupling constants.

We define the running coefficients as usual,

$$\frac{1}{\alpha_i(E_1)} = \frac{1}{\alpha_i(E_2)} - \frac{b_i}{2\pi} \ln \frac{E_2}{E_1} \,, \tag{13}$$

where $i = 1, 2, 3$ stands for the normalized hypercharge, $SU(2)_L$ and $SU(3)_C$ gauge couplings, respectively.

For the multiscale case, which we are interested in, eqs. (13) become for the chain **(a)**

$$\frac{2\pi}{\alpha_i(M_Z)} = \frac{2\pi}{\alpha_U} - [b_i^{(1)} ln \frac{M_S}{M_Z} + b_i^{(2)} ln \frac{M_2}{M_S} + b_i^{(3)} ln \frac{M_1}{M_2} + \\ b_i^{(4)} ln \frac{M_R}{M_1} + b_i^{(5)} ln \frac{M_C}{M_R} + b_i^{(6)} ln \frac{M_X}{M_C}] \,, \tag{14}$$

where $M_1 \equiv Max[M_R^2/M_C, M_C^2/M_X]$, $M_2 \equiv M_R^2/M_X$; for case **(b)** one has to interchange M_R and M_C in (14) as well as in the definitions of M_1 and M_2.

Let us study the cases **(a)** and **(b)** defined in (7) separately.

(a) $SO(10) \stackrel{M_X}{\rightarrow} SU(2)_L \times SU(2)_R \times SU(4)_C \stackrel{M_C}{\rightarrow} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \stackrel{M_R}{\rightarrow} SU(2)_L \times U(1)_Y \times SU(3)_C$. The coefficients are

Energy range	$b_1^{(k)}$	$b_2^{(k)}$	$b_3^{(k)}$
$M_Z < E < M_S$	$-(41+n)/10$	$(19-n)/6$	7
$M_S < E < M_2$	$-33/5$	-1	3
$M_2 < E < M_1$	$-(57+3\epsilon)/5$	$-(1+\epsilon)$	3
$M_1 < E < M_R$	$-(57+3\epsilon)/5$	$-(1+\epsilon)$	0
$M_R < E < M_C$	$-(45+3\epsilon)/5$	$-(1+\epsilon)$	0
$M_C < E < M_X$	$-(191+3\epsilon)/5$	$-(41+\epsilon)$	-34

(15)

where ϵ counts the number of heavy Higgs doublets that kicks in at M_R^2/M_X and $(n+1)$ is the total number of Higgs doublets below the supersymmetry breaking scale M_S. The detailed equations that follow from (14) specific to these models, as functions of ϵ and n, are given in Appendix B. They can be solved to obtain the three mass scales M_X, M_C and M_R in terms of the unification coupling α_U. Consistency will determine M_1, which is the maximum of M_R^2/M_C and M_C^2/M_X. Notice that we have assumed in (15) that $M_1 < M_R$, which is not necessarily true. However equation (15) shows that only b_1 is sensitive to the scale M_R (all the other coefficients remain constant), while only b_3 can feel the other scale M_1. For this reason it is irrelevant for the form of the renormalization group equations which of the two scales is smaller.

We have performed the calculation for values of the coupling constants $\alpha_1(M_Z) = 0.01695$, $\alpha_2(M_Z) = 0.03382$ and $\alpha_3(M_Z) = 0.119 \pm 0.003$.

For the case with just one **10** ($\epsilon = 0$), the unification scale M_X turns out to be independent of α_U. M_1 is M_R^2/M_C in this case, and it is found, for $n = 0$

$$\begin{aligned}
\log\left(\frac{M_X}{GeV}\right) &= 16.20 \pm 0.07 - 0.14 \log\left(\frac{M_S}{TeV}\right) \\
\log\left(\frac{M_C}{GeV}\right) &= 14.43 \pm 0.07 - 0.25 \log\left(\frac{M_S}{TeV}\right) + 0.07\alpha_U^{-1} \\
\log\left(\frac{M_R}{GeV}\right) &= 13.71 \pm 0.12 - 0.18 \log\left(\frac{M_S}{TeV}\right) + 0.10\alpha_U^{-1}
\end{aligned} \tag{16}$$

The lowest values are obtained for larger α_3. With $M_S \sim 1$ TeV and $\alpha_U \sim 1/5$, we get

$$\log\left(\frac{M_C}{GeV}\right) \simeq 14.77 \pm 0.07 \quad \log\left(\frac{M_R}{GeV}\right) \simeq 14.21 \pm 0.12 \tag{17}$$

Adding an extra 10-dimensional Higgs ($\epsilon = 1$) spoils the independence of M_X. In this case $M_1 = M_C^2/M_X$. We get, for $n = 0$

$$\begin{aligned}
\log\left(\frac{M_X}{GeV}\right) &= 14.38 \pm 0.25 + 0.14 \log\left(\frac{M_S}{TeV}\right) + 0.09\alpha_U^{-1} \\
\log\left(\frac{M_C}{GeV}\right) &= 12.77 \pm 0.25 + 0.03 \log\left(\frac{M_S}{TeV}\right) + 0.15\alpha_U^{-1} \\
\log\left(\frac{M_R}{GeV}\right) &= 10.25 \pm 0.46 + 0.34 \log\left(\frac{M_S}{TeV}\right) + 0.28\alpha_U^{-1}
\end{aligned} \tag{18}$$

We have the interesting result of M_X being *lower* for lower intermediate scales M_R. The smallest possible value of $(\alpha_U)^{-1}$ is dictated by proton decay constraints, requiring $\log(M_X/GeV) \geq 15.5$. Again, raising the supersymmetry breaking scale has the effect of lowering the intermediate scales, and for M_S of order 1 TeV

$$\log\left(\frac{M_C}{GeV}\right) \simeq 14.63 \pm 0.17 \quad \log\left(\frac{M_R}{GeV}\right) \simeq 13.62 \pm 0.29 \tag{19}$$

The lowest possible value corresponds to $\alpha_3 = 0.122$, and gives $\alpha_U^{-1} = 9.5$.

This low value of M_R is very interesting for neutrino physics, and so we concentrate on this case, $\epsilon = 1$. Figure 1 shows that the three scales M_X, M_C, M_R increase as a function of $1/\alpha_U$, for $M_S = 1$ TeV. It is interesting to see that the requirement of proton stability ($M_X > 10^{15.5}$ GeV) guarantees the perturbativity of the unified coupling, $1/\alpha_U > 12$.

In Figure 2 we plot our findings for M_R and M_C as functions of M_S for a fixed value of $M_X = 10^{15.5}$ GeV. Notice that M_R is almost insensitive to M_S (and completely so for $n = 1$), unlike M_C. Of course, the value of $M_S \simeq 1$ TeV is physically most interesting. Notice that there is some sensitivity to the precise value of α_3.

(b) SO(10)$\stackrel{M_X}{\rightarrow}$SU(2)$_L\times$SU(2)$_R\times$ SU(4)$_C$ $\stackrel{M_R}{\rightarrow}$ SU(2)$_L\times$ U(1)$_R\times$SU(4)$_C$ $\stackrel{M_C}{\rightarrow}$ SU(2)$_L\times$ U(1)$_Y\times$ SU(3)$_C$.

The b coefficients are now (here ϵ is the number of bidoublets from 10 with mass M_R)

Energy range	$b_1^{(k)}$	$b_2^{(k)}$	$b_3^{(k)}$
$M_Z < E < M_S$	$-(41+n)/10$	$(19-n)/6$	7
$M_S < E < M_2$	$-33/5$	-1	3
$M_2 < E < M_1$	$-97/5$	-1	-2
$M_1 < E < M_C$	$-97/5$	-3	-2
$M_C < E < M_R$	$-81/5$	-3	0
$M_R < E < M_X$	$-(189+3\epsilon)/5$	$-(43+\epsilon)$	-30

(20)

It can be seen that the extra bidoublets do not play a significant role in this case, contributing only at scales above M_R. In the minimal case of $\epsilon = 0, n = 0$, we get $M_1 = M_C^2/M_R$, and

$$
\begin{aligned}
\log\left(\frac{M_X}{GeV}\right) &= 14.88 \pm 0.11 - 0.13 \log\left(\frac{M_S}{TeV}\right) + 0.06\alpha_U^{-1} \\
\log\left(\frac{M_R}{GeV}\right) &= 13.19 \pm 0.11 - 0.24 \log\left(\frac{M_S}{TeV}\right) + 0.12\alpha_U^{-1} \\
\log\left(\frac{M_C}{GeV}\right) &= 13.04 \pm 0.14 - 0.17 \log\left(\frac{M_S}{TeV}\right) + 0.13\alpha_U^{-1}
\end{aligned} \tag{21}
$$

Again, lower intermediate scales are obtained for lower M_X. With $M_X \sim 10^{15.5}$ GeV,in this case, a smaller value of M_S is preferred. With $M_S \sim M_Z$,

$$
\log\left(\frac{M_R}{GeV}\right) \simeq 14.47 \pm 0.12 \quad \log\left(\frac{M_C}{GeV}\right) \simeq 14.35 \pm 0.12 \tag{22}
$$

the lowest value obtained for smaller α_s,and α_U is now 8.7. The case with the extra bidoublet gives essentially the same results.

What we observed here is that there can be a separation of all the scales involved, although sometimes not very big. The presence of extra light states (in addition to those of MSSM) below M_X, especially those that have non-trivial hypercharge, tended to spoil the succesful unification for too big separation of scales. However, as shown in (18), it is perfectly possible to push scales to the low intermediate range. Finally we remind that the results presented here are only a rough guide, since we have not included the two-loop and threshold effects.

V. PHENOMENOLOGICAL AND COSMOLOGICAL CONSEQUENCES

We now turn to the discussion of the resulting physics of the theory. The most important features are the exact R-parity at low energies, i.e. the MSSM as an effective low-energy theory and the "quasi-canonical" see-saw form with a rather light neutrino.

i) R-parity. Here the situation parallels the one already found by some of us in the context of LR symmetric theories [14]. We showed that at the large scale $\langle 16 \rangle = \langle \tilde{\nu}_c \rangle = 0$ and so R-parity is conserved. Next, as in any theory based on the renormalizable see-saw mechanism [5], one can show that $\langle \tilde{\nu} \rangle = 0$ too. This is valid at all energies all the way to the electroweak scale and below (of course one has to assume $\langle\tilde{\nu}\rangle = 0$ at high scale, as with other fields that break the SM symmetry).

It is worth repeating this simple, but important argument. It is well-known that in the MSSM we cannot break R-parity spontaneously. The problem is this would imply the existence of the Majoron J, since this also breaks lepton number spontaneously [15], and the Z boson would then have an extra decay into a Majoron and its real partner.

Now, after we integrate the heavy fields out, we are left effectively with the MSSM: at low energies all the effects of the new scale M_R must go as $1/M_R$. The same is true of the would-have-been Majoron J: its mass must be suppressed by $1/M_R$ and Z can still decay into $J + partner$. Again, as in the MSSM, this is ruled out and we learn that R-parity can never be broken. This is an extremely important prediction, since then the LSP is absolutely stable.

ii) Neutrino Mass. One of the main motivations behind the SO(10) unification is the natural implementation of the see-saw mechanism. We chose **126** and $\overline{\mathbf{126}}$ Higgs multiplets in order to have a renormalizable see-saw which then gave us R-parity to be exact all the way down to the MSSM.

The first thing we notice is that for both patterns of symmetry breaking the allowed value for M_R is pretty high (usually about 10^{14} GeV or higher, although it can go down to 10^{12} GeV, as in (18)). Second, supersymmetry helps us not to have uncontrollable VEVs from the left-handed triplet VEVs, and indeed the supersymmetry breaking effects are negligible for such high M_R [5].

Further, as we emphasized in [5], the exact form of the see-saw is rather model dependent in supersymmetric SO(10) theories. The canonical form for the see-saw for our model is spoiled by effective non-renormalizable terms in the superpotential of the form

$$\frac{1}{M_X}\bar{\Delta}\Phi^2\bar{\Delta}_c \tag{23}$$

(where $\bar{\Delta}$ and $\bar{\Delta}_c$ are left and right-handed triplets respectively, from $\overline{\mathbf{126}}$ and Φ is a bidoublet from **10**) which arise once the heavy **54** fields get integrated out. Such terms then produce nonvanishing VEVs for the left-handed triplets since they get generated from the term above with $m_\Sigma\Delta\overline{\Delta}$ to give

$$\langle\bar{\Delta}\rangle = \frac{\langle\bar{\Delta}_c\rangle}{M_X m_\Sigma}\langle\Phi\rangle^2 \equiv \epsilon\frac{\langle\Phi\rangle^2}{\langle\bar{\Delta}_c\rangle} \tag{24}$$

where $\epsilon \equiv \langle\bar{\Delta}_c\rangle^2/M_X m_\Sigma$ can be anywhere between 1 and 10^{-4} in the above chains of symmetry breaking.

Thus the formula for neutrino masses is

$$m_\nu \approx (f^2\epsilon - h_D^2)\frac{\langle\Phi\rangle^2}{m_{\nu_R}} \tag{25}$$

where f is the coupling of triplets and right-handed neutrinos.

We see now that the ϵ term can not be neglected compared to the canonical term coming from the neutrino Dirac masses, at least for the lighter generations. This might be a welcome addition to the otherwise rather constrained scenarios [16].

Of course, in some specific models one can have this new non-canonical term in the see-saw formula vanishing. For example, this is what happens if the Dimopoulos-Wilczek mechanism [17] is employed to solve the doublet-triplet splitting problem. Let us briefly repeat the argument [5]. As mentioned above, the form (23) is obtained from the terms (H is the 10-dimensional Higgs)

$$\lambda_S S\Sigma^2 + \lambda_H S H^2 + \frac{m_S}{2} S^2 \tag{26}$$

after integrating out the heavy S, for which the presence of the term $\lambda_H S H^2$ is crucial. However, to solve the doublet-triplet problem this same term must be absent, which is obtained for example with an additional Z_3 symmetry [18]. The Dimopoulos-Wilczek mechanism is then implemented by the new terms (among others)

$$\lambda_{H'} S H'^2 + \lambda_{HH'} H A' H' \ , \tag{27}$$

where H and H' are **10**'s and A' a new **45** with the vev $< A' >= diag(a', a', a', 0, 0) \times \tau_2$.

Since the $\lambda_H S H^2$ term is forbidden, only the $\mathrm{SU}(2)_L$ doublets from H (but not from H') have zero mass. So the Dimopoulos-Wilczek mechanism at the same time solves the doublet-triplet splitting problem and gives the canonical see-saw. The price to pay is however the ad-hoc addition of a discrete symmetry and the inclusion of more extra fields.

iii) Proton decay and related issues. The usual lore is that the unification constraints favor the minimal supersymmetric SU(5) grand unified theory. One gets the single scale GUT breaking $M_X \simeq 2 \times 10^{16}$ GeV, which makes the usual $d = 6$ contribution to proton decay quite small:

$$\tau(p \to \pi^0 e^+) \geq 10^{36 \pm 1.5} yr. \tag{28}$$

and the dominant decay mode $p \to K^+ \bar{\nu}$ comes from the $d = 5$ operators [19]. It is also known that in SO(10) models, due to misalignment of the Higgs triplet and doublet Yukawa couplings and depending on the details of the underlying flavor theory, other decay modes to kaons may be dominant, such as $p \to K^0 l^+$ [20].

However, what we see in the types of models studied here, is that unification through alternate symmetry breaking channels is quite possible. Moreover even the "smoking gun" signature of supersymmetric models with kaon production in proton decay, may not be dominant. Namely, we saw that, contrary to the common belief, lowering of the intermediate scale M_R can cause *lowering* of the grand unified scale (at least in the case with more than one 10 Higgs multiplet). This then opens the possibility that the gauge boson mediated proton decay may be the dominant one and thus we could even have the possibility where in a supersymmetric model the dominant decay products be pions instead of kaons [1]!

Let us also comment on other higher dimensional operators. We know that the dangerous $d = 4$ proton decay is absent, since R-parity is exact even at $E < M_W$. Of course, R-parity

---

[1] An important, although not dominant contribution of the $p \to \pi^0 e^+$ mode has been found in [21], but there it was produced by $d = 5$ operators.

cannot guarantee the absence of higher dimensional operators which break the baryon and lepton number. Since $\mathbf{16}^4$ is invariant under SO(10), one can have

$$\Delta W_{effective} = C\frac{QQQL}{M} + ... \tag{29}$$

Even if $M = M_{Pl}$, unless C is extremely small, the above interaction would be dangerously large. Of course, there is no proof that it must be present, but still, it would be nice to have some explanation for its absence.

If one dislikes ad-hoc discrete symmetries, as we do (in spite of the euphemism that they can come from superstrings), one natural possibility is to go to $E(6)$ GUT. In $E(6)$ the basic representation is $27 = 16_{SO(10)} + 10_{SO(10)} + 1_{SO(10)}$, and the dangerous interaction $(27)^4$ is not allowed. Of course, one must then split naturally the Higgs in $10_{SO(10)}$ from the new states D and D^c (down-quark like) in 10. The study of this is beyond the scope of this work and is related to the construction of the $E(6)$ grand unified theory.

Of course, even without going beyond SO(10) or including nonrenormalizable operators, one gets proton decay mediating $d = 5$ operators (29) as usual after integrating out the heavy color triplets and antitriplets, i.e. $(1, 1, 6)$ from $\mathbf{10}$ (the same contribution as in SU(5)), but also from $\overline{\mathbf{126}}$ [21].

VI. SUMMARY

There are a number of well-known reasons that make SO(10) a popular grand unified theory. The two main ones are the grouping of families of fermions in 16-dimensional spinor representations, and the natural incorporation of the see-saw mechanism. The supersymmetric extension of the theory provides yet another important rationale, namely, the otherwise *ad hoc* discrete R-parity symmetry of the MSSM becomes a finite SO(10) gauge transformation. As we showed in this paper, the symmetry remains exact throughout all the stages of symmetry breaking down to the electroweak scale. Thus, the low energy effective theory is completely determined: it is the MSSM with absolutely stable LSP. This result is a general property of the supersymmetric see-saw mechanism and a spontaneously broken B-L symmetry [5].

An important phenomenological prediction of the theory regards the proton lifetime. Remarkably enough, the theory allows for an intermediate see-saw scale at the expense of lowering the unification scale compared to the single-step breaking of supersymmetric GUTs. As argued in [6], the existence of intermediate scales is due to the violation of the so-called survival principle, i.e. to the fact that there is a number of supermultiplets whose masses lie below the corresponding scales of symmetry breaking. The typical value of their masses is $\sim M_R^2/M_X$. We find that the lowest possible values of M_R are of order 10^{13-14}GeV, and with $M_X \sim 10^{16}$ GeV, these states turn out to be out of experimental reach.

Contrary to the usual belief, the low unification scale allows for the interesting possibility of proton decay being dominated by the usual $d = 6$ mode characteristic of ordinary GUTs, $p \to e^+ \pi^0$. Now, this is tied up, as we said, to an intermediate see-saw scale in the range 10^{13-14}GeV. This is encouraging for the neutrino mass expectations. However, we must admit that (unlike the often present claims in the literature), one cannot really predict neutrino masses. This is clear from the central formula (25), which shows that besides the usual canonical term proportional to the squared Dirac Yukawa coupling, there is another piece which cannot be predicted by the SO(10) symmetry itself. Clearly, there is enough freedom to accomodate the solar and atmospheric neutrino data, but no way of making honest predictions.

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APPENDIX A

We will denote SO(10) indices by $i, j, ..$, SO(6) indices by $a, b, ...$, and SO(4) indices by $\alpha, \beta, ...$ With this convention, the field S in the **54** representation is represented by the symmetric, traceless second order tensor S_{ij}; the field in the **45** representation, A, by the antisymmetric second order A_{ij}; and the fields Σ and $\overline{\Sigma}$ in the **126** and $\overline{\mathbf{126}}$ representations by the fifth-order antisymmetric tensors Σ_{ijklm} and $\overline{\Sigma}_{ijklm}$, with self- and anti-self-duality conditions

$$\Sigma_{ijklm} = \frac{i}{5!}\epsilon_{ijklmopqrs}\Sigma_{opqrs} \quad , \quad \overline{\Sigma}_{ijklm} = -\frac{i}{5!}\epsilon_{ijklmopqrs}\overline{\Sigma}_{opqrs} \quad .$$

The superpotential is

$$\begin{aligned} W = &\frac{m_S}{2}\mathrm{Tr}S^2 + \frac{\lambda_S}{3}\mathrm{TrS}^3 + \frac{m_A}{2}\mathrm{Tr}A^2 + \lambda \mathrm{Tr}A^2 S \\ &+ m_\Sigma \Sigma\overline{\Sigma} + \eta_S \Sigma^2 S + \overline{\eta}_s \overline{\Sigma}^2 S + \eta_A \Sigma\overline{\Sigma}A \quad , \end{aligned} \tag{30}$$

where

$$\Sigma\overline{\Sigma}A = \frac{1}{4!}\Sigma_{ijklm}\overline{\Sigma}_{ijklp}A_{mp} \quad , \quad \Sigma\overline{\Sigma} = \Sigma_{ijklm}\overline{\Sigma}_{ijklm} \quad , \quad \Sigma^2 S = \frac{1}{4!}\Sigma_{ijklm}\Sigma_{ijklp}S_{mp} \quad .$$

The F-terms are then

$$(F_A)_{ji} = m_A A_{ij} + \lambda(A_{il}S_{lj} + S_{il}A_{lj}) + \frac{\eta_A}{4!}[\Sigma_{klmpj}\overline{\Sigma}_{klmpi} - \Sigma_{klmpi}\overline{\Sigma}_{klmpj}] \, ,$$

$$\begin{aligned} (F_S)_{ij} = &m_S S_{ij} + \lambda_S[(S^2)_{ij} - \frac{\delta_{ij}}{10}\mathrm{Tr}S^2] + \lambda[(A^2)_{ij} - \frac{\delta_{ij}}{10}\mathrm{Tr}A^2] \\ &+ \frac{\eta_S}{4!}\Sigma_{klmpi}\Sigma_{klmpj} + \frac{\overline{\eta}_S}{4!}\overline{\Sigma}_{klmpi}\overline{\Sigma}_{klmpj} \, , \end{aligned}$$

$$(F_{\overline{\Sigma}})_{ijklm} = m_\Sigma \Sigma_{ijklm} + \eta_A[A_{ip}\Sigma_{pjklm} + \frac{i}{4!}\epsilon_{ijklmopqrs}A_{ot}\Sigma_{tpqrs}]$$

$$+ 2\overline{\eta}_S[S_{ip}\overline{\Sigma}_{pjklm} + \frac{i}{4!}\epsilon_{ijklmopqrs}S_{ot}\overline{\Sigma}_{tpqrs}] \ ,$$

$$(F_\Sigma)_{ijklm} = m_\Sigma\overline{\Sigma}_{ijklm} + \eta_A[A_{ip}\overline{\Sigma}_{pjklm} - \frac{i}{4!}\epsilon_{ijklmopqrs}A_{ot}\overline{\Sigma}_{tpqrs}]$$
$$+ 2\eta_S[S_{ip}\Sigma_{pjklm} - \frac{i}{4!}\epsilon_{ijklmopqrs}S_{ot}\Sigma_{tpqrs}] \ . \qquad (31)$$

The VEVs for S and A that ensure the required symmetry breaking SO(10) $\rightarrow$ SU(2)$_L\times$SU(2)$_R\times$SU(4)$_C$ are

$$S = s\,\mathrm{diag}(1,1,1,-3/2,-3/2)\times 1_2 \quad , \quad A = \mathrm{diag}(a,a,a,b,b)\times\tau_2 \quad . \qquad (32)$$

The VEV of Σ and $\overline{\Sigma}$ are a bit more complicated. In the basis where the Cartan generators are diagonal, the SU(2)$_L\times$U(1)$_Y\times$SU(3)$_C$ singlets in Σ and $\overline{\Sigma}$ get VEVs:

$$\langle\Sigma'_{13579}\rangle = \sigma \quad , \quad \langle\overline{\Sigma}'_{246810}\rangle = \bar{\sigma} \quad . \qquad (33)$$

One then goes to the basis in which (32) is valid, to find the 2^5 components of Σ and $\overline{\Sigma}$ that get a nonvanishing VEV. They have the first index equal to 1 or 2; the second to 3 or 4, etc. Each have a VEV $2^{-5/2}\sigma$, with a factor of $-i$ for each even index in Σ and a factor of $+i$ for each even index in $\overline{\Sigma}$.

The VEV directions are chosen to get the required symmetry breaking, and of course are not the only possible ones. To make sure that the vacuum chosen is not connected to the undesired ones by flat directions, one has to examine the mass spectrum: the flat directions will be associated with massless excitations. It is clear from the analysis in Section III that no such states exist for our vacuum.

The VEV equations are then written as

$$\sigma[m_\sigma + \eta_A(3a+2b)] = \bar{\sigma}[m_\sigma + \eta_A(3a+2b)] = 0 \qquad (34)$$

$$(m_A + 2\lambda s)a + \frac{\eta_A}{2}\sigma\bar{\sigma} = 0 \qquad (35)$$

$$(m_A - 3\lambda s)b + \frac{\eta_A}{2}\sigma\bar{\sigma} = 0 \qquad (36)$$

$$s[m_S - \frac{\lambda_S}{2}s] + \frac{2}{5}\lambda(a^2 - b^2) = 0 \qquad (37)$$

Then if one fine-tunes $(m_A + 2\lambda s)a \sim M_R^2$, the first pattern **(a)** of symmetry breaking is obtained

$$SO(10) \xrightarrow{\langle S\rangle} SU(2)_L\times SU(2)_R\times SU(4)_C \xrightarrow{\langle A\rangle} SU(2)_L\times SU(2)_R\times SU(3)_C\times U(1)_{BL}$$
$$\xrightarrow{\langle\Sigma\rangle} SU(2)_L\times SU(3)_C\times U(1)_Y \qquad (38)$$

with

$$M_X = s\sim m_S\sim m_A\ ; \quad M_C = a\sim m_\Sigma\ ; \quad M_R^2 = \sigma\bar{\sigma} \qquad (39)$$

and $b \sim M_R^2/M_X$.

The second pattern **(b)** is obtained by instead fine-tuning $(m_A - 3\lambda s)a \sim M_R^2$. Then

$$\begin{aligned} SO(10) &\xrightarrow{\langle S\rangle} SU(2)_L \times SU(2)_R \times SU(4)_C \xrightarrow{\langle\Sigma,\overline{\Sigma}\rangle} SU(2)_L \times U(1)_R \times SU(4)_C \\ &\xrightarrow{\langle A\rangle} SU(2)_L \times SU(3)_C \times U(1)_Y \end{aligned} \tag{40}$$

$$M_X = s \sim m_S \sim m_A \gg M_R^2 = \sigma\bar{\sigma} \gg \quad M_C = b \sim m_\Sigma \tag{41}$$

and $a \sim M_C^2/M_X$.

The contribution of the representations S(54), A(45), Σ(126) and $\overline{\Sigma}$ to the D-terms is as follows:

$$D_{ij} = (-ig_U)(2S^*_{k[i}S_{j]k} - 2A^*_{k[i}A_{j]k} + 5\Sigma^*_{klmn[i}\Sigma_{j]klmn} + 5\overline{\Sigma}^*_{klmn[i}\overline{\Sigma}_{j]klmn} + ...) \ . \tag{42}$$

The contributions of the two index representations S and A may be written as commutators

$$D_{ij} = (-ig_U)(2[S^\dagger, S] + 2[A^\dagger, A] + ...)_{ij} \ , \tag{43}$$

but then eq. (32) for the vevs of the matrices S and A immediately give zero for their contribution to the D terms.

Thus, it remains to see what the $\mathbf{126}, \overline{\mathbf{126}}$ representations give. It is most convenient to work in the complex basis where the Cartan generators are diagonal (indices $A, B = 1, 2...10$) which is obtained from the usual Cartesian basis ($i, j = 1, 2...10$) by the unitary transformation $U = 1_5 \times U_2$, where $U_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix}$ by $V_A = U_{Ai}V_i$.

It is easy to see that in this basis the $\mathbf{126}$ and $\overline{\mathbf{126}}$ part of $D_{AB} = U_{Ai}U_{Bj}D_{ij}$ is

$$D_{AB} = (-5ig_U)\left(\Sigma^*_{GCDEF}g_{G[A}\Sigma_{B]CDEF} + \overline{\Sigma}^*_{GCDEF}g_{G[A}\overline{\Sigma}_{B]CDEF}\right) = D^1_{AB} - D^1_{BA} \ , \tag{44}$$

where g_{AB} is the metric in the complex basis $g = 1_5 \times \tau_1$ and

$$D^1_{AB} = (-5ig_U)\left(\Sigma^*_{GCDEF}g_{GA}\Sigma_{BCDEF} - \overline{\Sigma}^*_{GCDEF}g_{GB}\overline{\Sigma}_{ACDEF}\right) \tag{45}$$

can have non-vanishing vev (when the vevs of $\Sigma, \overline{\Sigma}$ take the form of eq. (33)) only if A,B form an even-odd conjugate pair of indices. Thus, without loss of generality, we have for A even and B odd

$$< D_{AB} >= -ig_U(5!)g_{AB}(|\sigma|^2 - |\bar{\sigma}|^2) + ... \ . \tag{46}$$

It follows that provided the magnitudes of $\sigma, \bar{\sigma}$ are equal, the D terms vanish at the first two stages of symmetry breaking specified by the vevs in eqs. (32), (33).

The absence of flat directions characterized by holomorphic invariants formed from the fields involved in the high scale symmetry breaking $(S, A, \Sigma, \overline{\Sigma})$ *alone* then follows *post-hoc* from the calculated spectra of these fields which do not include any massless modes (aside from those eaten by massive gauge bosons). The usual argument [22,14] serves to invoke the protection of the soft terms against charge and color breaking flat directions involving the light field vevs.

APPENDIX B

We now give in detail the renormalization group equations for each symmetry-breaking pattern.

Case (a) $M_2 = M_R^2/M_X$.

For M_1 there are two possibilities:

1) $M_1 = M_R^2/M_C$

Then (14) becomes

$$\frac{2\pi}{\alpha_i(M_Z)} = \frac{2\pi}{\alpha_U} - [(b_i^{(1)} - b_i^{(2)})S + (2b_i^{(2)} - b_i^{(4)} - b_i^{(5)})R + (-b_i^{(3)} + b_i^{(4)} + b_i^{(5)} - b_i^{(6)})C + (-b_i^{(2)} + b_i^{(3)} + b_i^{(6)})X] \,, \qquad (47)$$

where $Y = ln(M_Y/M_Z)$ for any scale M_Y. Using (15) we get

$$\frac{2\pi}{\alpha_1(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{10}S + \frac{36+6\epsilon}{5}R + \frac{146}{5}C - \frac{215+6\epsilon}{5}X\right) \,, \qquad (48)$$

$$\frac{2\pi}{\alpha_2(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{6}S + (2\epsilon)R + 40C - (41+2\epsilon)X\right) \,, \qquad (49)$$

$$\frac{2\pi}{\alpha_3(M_Z)} = \frac{2\pi}{\alpha_U} - (4S + 6R + 31C - 34X) \;. \qquad (50)$$

2) $M_1 = M_C^2/M_X$

In this case (14) becomes

$$\frac{2\pi}{\alpha_i(M_Z)} = \frac{2\pi}{\alpha_U} - [(b_i^{(1)} - b_i^{(2)})S + (2b_i^{(2)} - 2b_i^{(3)} + b_i^{(4)} - b_i^{(5)})R + (2b_i^{(3)} - 2b_i^{(4)} + b_i^{(5)} - b_i^{(6)})C + (-b_i^{(2)} + b_i^{(4)} + b_i^{(6)})X] \,, \qquad (51)$$

and with numbers

$$\frac{2\pi}{\alpha_1(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{10}S + \frac{36+6\epsilon}{5}R + \frac{146}{5}C - \frac{215+6\epsilon}{5}X\right) \,, \qquad (52)$$

$$\frac{2\pi}{\alpha_2(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{6}S + (2\epsilon)R + 40C - (41+2\epsilon)X\right) \,, \qquad (53)$$

$$\frac{2\pi}{\alpha_3(M_Z)} = \frac{2\pi}{\alpha_U} - (4S + 40C - 37X) \;. \qquad (54)$$

Case (b) $M_2 = M_C^2/M_X$.

We get the equivalent equations as (14), (47) and (51) just interchanging R and C. We have again two cases:

1) $M_1 = M_C^2/M_R$

$$\frac{2\pi}{\alpha_1(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{10}S + \frac{112}{5}C + \frac{108+3\epsilon}{5}R - \frac{253+3\epsilon}{5}X\right) \ , \tag{55}$$

$$\frac{2\pi}{\alpha_2(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{6}S + 4C + (38+\epsilon)R - (43+\epsilon)X\right) \ , \tag{56}$$

$$\frac{2\pi}{\alpha_3(M_Z)} = \frac{2\pi}{\alpha_U} - (4S + 8C + 30R - 35X) \ . \tag{57}$$

2) $M_1 = M_R^2/M_X$

$$\frac{2\pi}{\alpha_1(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{10}S + \frac{112}{5}C + \frac{108+3\epsilon}{5}R - \frac{253+3\epsilon}{5}X\right) \ , \tag{58}$$

$$\frac{2\pi}{\alpha_2(M_Z)} = \frac{2\pi}{\alpha_U} - \left(\frac{25-n}{6}S + (44+\epsilon)R - (45+\epsilon)X\right) \ , \tag{59}$$

$$\frac{2\pi}{\alpha_3(M_Z)} = \frac{2\pi}{\alpha_U} - (4S + 8C + 30R - 35X) \ . \tag{60}$$

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FIGURES

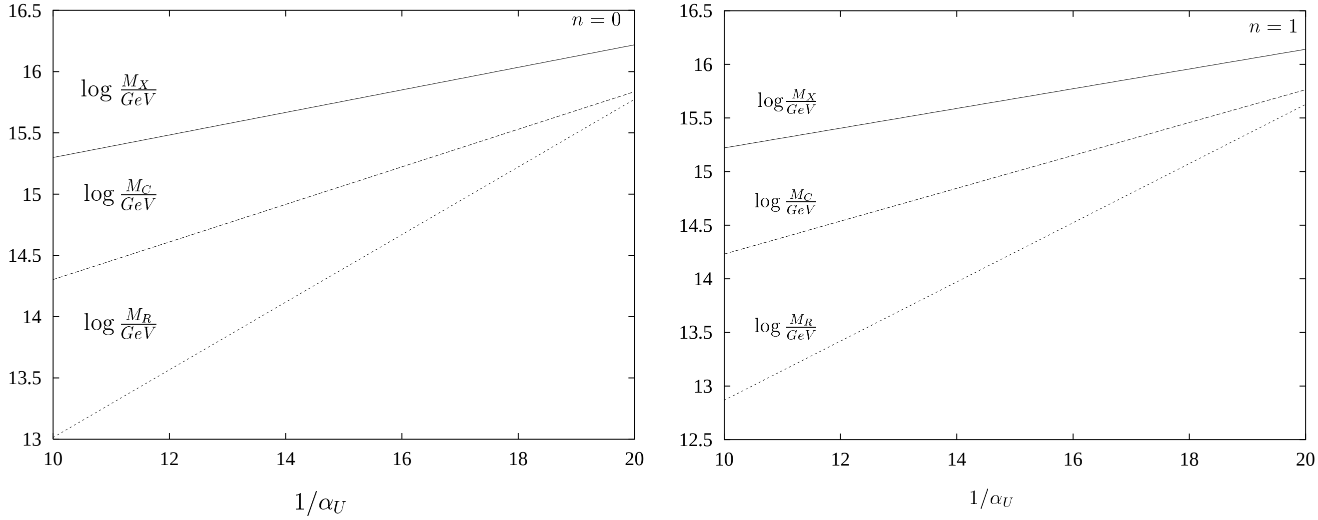

FIG. 1. M_C, M_R and M_X vs. $1/\alpha_U$ for Chain A, with $M_s = 1$ TeV and $\epsilon = 1$, for $n = 0$ and 1.

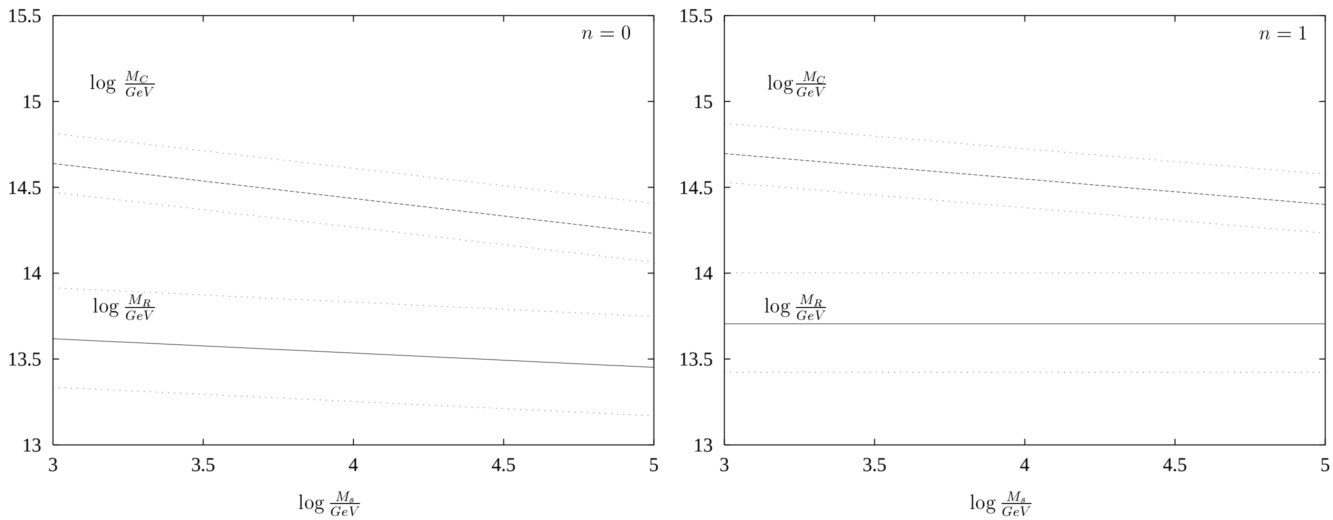

FIG. 2. M_C and M_R vs. M_s for Chain A, when M_X is fixed at its lowest allowed value ($log(M_X/GeV) = 15.5$) and $\epsilon = 1$, for $n = 0$ and 1. Dotted lines are the same plots for lowest and highest values of α_3

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