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WEAK COMPACTNESS OF UNCONDITIONALLY CONVERGENT OPERATORS ON $C_o(T)$

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Abstract

Let T be a locally compact Hausdorff space and let $C_o(T)$ be the Banach space of all complex valued continuous functions vanishing at infinity in T, provided with the supremum norm. Let X be a locally convex Hausdorff space (briefly, an lcHs) which is quasicomplete. By using Rosenthal's lemma it is shown that every lcHs-valued unconditionally convergent operator on $C_o(T)$ is weakly compact. Then it is deduced that every continuous linear map $u : C_o(T) \to X$ is weakly compact if $c_o \not\subset X$.

1. INTRODUCTION

Let T be a locally compact Hausdorff space and $C_o(T)$ the Banach space of all complex valued continuous functions vanishing at infinity in T, endowed with the supremum norm.

If X is a Banach space and K is a compact Hausdorff space, then Pelczyński [11] showed that every X-valued unconditionally convergent operator on $C(K)$ is weakly compact. This result was extended in Theorem 12 of [9] to unconditionally convergent continuous linear maps on $C_o(T)$ with values in a locally convex Hausdorff space (briefly, an lcHs) which is quasicomplete. In this note, adapting the proof of Theorem VI.2.15 of Diestel and Uhl [1] in which Rosenthal's lemma plays a key role, we give a simple measure theoretic proof of Theorem 12 of [9] and then deduce the result of Thomas [12] to the effect that every continuous linear map u on $C_o(T)$ with values in a quasicomplete lcHs X is weakly compact whenever $c_o \not\subset X$. The Banach space analogue of the latter result is due to Pelczyński [10].

2. PRELIMINARIES

In this section we fix notation and terminology. For the convenience of the reader we also give some definitions and results from the literature.

In the sequel T will denote a locally compact Hausdorff space and $C_o(T)$ the Banach space of all complex valued continuous functions vanishing at infinity in T, endowed with the supremum norm $||f||_T = \sup_{t \in T} |f(t)|$.

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DEFINITION 1. Let $\mathcal{B}(T)$ be the σ-algebra of the Borel sets of T. A complex measure μ on $\mathcal{B}(T)$ is said to be *Borel-regular* (resp. *Borel-outer regular*) if, given $E \in \mathcal{B}(T)$ and $\epsilon > 0$, there exist a compact K and an open set U in T with $K \subset E \subset U$ (resp. an open set U in T) with $E \subset U$ such that $|\mu(B)| < \epsilon$ for every $B \in \mathcal{B}(T)$ with $B \subset U \backslash K$ (resp. $B \subset U \backslash E$).

$M(T)$ is the Banach space of all bounded complex Radon measures on T with their domain restricted to $\mathcal{B}(T)$ so that each $\mu \in M(T)$ is a regular (bounded) complex Borel measure on T and has norm $||\cdot||$ given by $||\mu|| = var(\mu, T)$ where the variation of μ is taken with respect to $\mathcal{B}(T)$. We denote $var(\mu, E)$ by $|\mu|(E)$, for $E \in \mathcal{B}(T)$.

A vector measure is an additive set function defined on a ring of sets with values in an lcHs. In the sequel X denotes an lcHs with topology τ. Γ is the set of all τ-continuous seminorms on X. The dual of X is denoted by X^*.

The strong topology $\beta(X^*, X)$ of X^* is the locally convex topology induced by the seminorms $\{p_B : B \text{ bounded in } X\}$, where $p_B(x^*) = \sup_{x \in B} |x^*(x)|$. X^{**} denotes the dual of $(X^*, \beta(X^*, X))$ and is endowed with the locally convex toplogy τ_e of uniform convergence on equicontinuous subsets of X^*. Note that $(X^*, \beta(X^*, X))$ and (X^{**}, τ_e) are lcHs.

It is well known that the canonical injection $J : X \to X^{**}$ given by $< Jx, x^* >=< x, x^* >$ for all $x \in X$ and $x^* \in X^*$, is linear. On identifying X with $JX \subset X^{**}$, one has $\tau_e|_{JX} = \tau_e|_X = \tau$.

Let $\mathcal{E} = \{A \subset X^* : A \text{ is equicontinuous }\}$. Then the family of seminorms $\Gamma_{\mathcal{E}} = \{p_A : A \in \mathcal{E}\}$ induces the topology τ of X and the topology τ_e of X^{**}, where $p_A(x) = \sup_{x^* \in A} |x^*(x)|$ for $x \in X$ and $p_A(x^{**}) = \sup_{x^* \in A} |x^{**}(x^*)|$ for $x^{**} \in X^{**}$.

DEFINITION 2. A linear map $u : C_o(T) \to X$ is called a *weakly compact operator* on $C_o(T)$ if $\{uf : ||f||_T \leq 1\}$ is relatively weakly compact in X.

The following result is the same as Lemma 2 of [9], where the hypothesis of quasicompleteness of X is redundant.

PROPOSITION 1. *Let X be an lcHs and let $u : C_o(T) \to X$ be a continuous linear map. Then u^*A is bounded in $M(T)$ for each $A \in \mathcal{E}$.*

The following result (Corollary 9.3.2 of Edwards [3] which is essentially due to Lemma 1 of Grothendieck [4]) plays a key role in Section 3.

PROPOSITION 2. *Let E and F be lcHs with F quasicomplete and let $u : E \to F$ be linear and continuous. Then the following assertions are equivalent:*

(i) $u^{**}(E^{**}) \subset F$.

(ii) u maps bounded subsets of E into relatively weakly compact subsets of F.

(iii) $u^*(A)$ *is relatively* $\sigma(E^*, E^{**})$*-compact for each equicontinuous subset* A *of* F^*.

The following result is due to Theorem 2 of Grothendieck [4], and is needed in Section 3.

PROPOSITION 3. *A bounded set* A *in* $M(T)$ *is relatively weakly compact if and only if, for each disjoint sequence* $\{U_n\}_1^\infty$ *of open sets in* T,

$$\sup_{\mu \in A} |\mu(U_n)| \to 0$$

as $n \to \infty$.

For each τ-continuous seminorm p on X, let $p(x) = ||x||_p$, $x \in X$, and let $X_p = (X, ||\cdot||_p)$ be the associated seminormed space. The completion of the quotient normed space $X_p/p^{-1}(0)$ is denoted by $\tilde{X}_p$. Let $\Pi_p : X_p \to X_p/p^{-1}(0) \subset \tilde{X}_p$ be the canonical quotient map.

Let $\mathcal{S}$ be a σ-algebra of subsets of a non empty set Ω. An X-valued vector measure m on $\mathcal{S}$ is said to be *bounded* if $\{m(E) : E \in \mathcal{S}\}$ is bounded in X.

For the theory of integration of bounded $\mathcal{S}$-measurable scalar functions with respect to a bounded quasicomplete lcHs-valued vector measure defined on the σ-algebra $\mathcal{S}$, the reader may refer to [7] or [9]. We need the following results from Lemma 6 of [7] and Proposition 7 of [9].

PROPOSITION 4. *Let* X *be a quasicomplete lcHs and let* $\mathcal{S}$ *be a* σ*-algebra of subsets of* Ω. *Then:*

(i) If f *is a bounded* $\mathcal{S}$*-measurable scalar function and* m *is an* X*-valued bounded vector measure on* $\mathcal{S}$, *then* f *is* m*-integrable in* Ω *and*

$$x^*\left(\int_\Omega f dm\right) = \int_\Omega f d(x^* \circ m)$$

for each $x^* \in X^*$.

(ii) (Lebesgue bounded convergence theorem) If m *is an* X*-valued* σ*-additive vector measure on* $\mathcal{S}$ *and* (f_n) *is a bounded sequence of* $\mathcal{S}$*-measurable scalar functions with* $\lim_n f_n(w) = f(w)$ *for each* $w \in \Omega$, *then* f *is* m*-integrable in each* $E \in \mathcal{S}$ *and*

$$\int_E f dm = \lim_n \int_E f_n dm$$

for each $E \in \mathcal{S}$.

The following result is due to the first part of Theorem 1 of [9], which is analogous to Theorem VI.2.1 of [1], for lcHs-valued continuous linear maps on $C_o(T)$. It plays a vital role in Section 3.

PROPOSITION 5. *Let* X *be an lcHs and let* $u : C_o(T) \to X$ *be a continuous linear map. Then there exists a unique* X^{**}*- valued vector measure* m *on* $\mathcal{B}(T)$ *satisfying the following properties:*

(i) $(x^* \circ m) \in M(T)$ *for each* $x^* \in X^*$ *and consequently,* $m : \mathcal{B}(T) \to X^{**}$ *is* σ*-additive in* $\sigma(X^{**}, X^*)$*-topology.*

(ii) *The mapping* $x^* \to x^* \circ m$ *of* X^* *into* $M(T)$ *is weak*-weak* continuous. Moreover,* $u^*x^* = x^* \circ m, \; x^* \in X^*$.

(iii) $x^*uf = \int_T f d(x^* \circ m)$ *for each* $f \in C_o(T)$ *and* $x^* \in X^*$.

(iv) *The range of* m *is* τ_e*-bounded in* X^{**}.

(v) $m(E) = u^{**}(\chi_E)$ *for* $E \in \mathcal{B}(T)$.

(vi) *If* X *is quasicomplete, then by (iii) and (iv) and by Proposition 4(i),* $uf = \int_T f dm$ *for* $f \in C_o(T)$.

DEFINITION 3. Let $u : C_o(T) \to X$ be a continuous linear map. Then the vector measure m as given in Proposition 5 is called the *representing measure* of u.

DEFINITION 4. Let X and Y be quasicomplete lcHs and let $u : X \to Y$ be a continuous linear map. Then u is called an unconditionally convergent operator if for every unconditonally weakly Cauchy series $\sum_1^\infty x_n$ in X (in the sense that $(x_n)_1^\infty \subset X$ with $\sum_1^\infty |x^*(x_n)| < \infty$ for all $x^* \in X^*$), the series $\sum_1^\infty u(x_n)$ is unconditionally convergent in Y.

3. MAIN THEOREM

In [11] Pelczyński proved that unconditionally convergent operators on $C(K)$, K a compact Hausdorff space, with values in a Banach space are weakly compact. This result was generalized to quasicomplete lcHs in Theorem 12 of [9], but its proof uses some deep results such as Theorem 1 of [8] (characterizations of weakly compact sets in $M(T)$ in terms of the Baire restrictions of the memebers of the set in question) and Theorem 3(vii) of [9] whose proof is also based on the former result of [8]. The aim of the present section is to provide an elementary proof of Theorem 12 of [9]. For this we generalize in Lemma 1 the first part of Theorem VI.2.15 of Diestel and Uhl [1] to quasicomplete lcHs. As in the original proof of [1] we use Theorem 2 of [4] and Rosenthal's lemma on p.18 of [1]. Using Lemma 1 and the fact that $C_o(T)$ has the strict Dunford-Pettis property (see Lemma 2) we deduce Theorem 1 which is the same as Theorem 12 of [15]. Thereafter, we obtain the first part of Theorem 5.3 of Thomas [12] by applying (i)⇒(ii) of Theorem 1 (and hence is independent of Lemma 2).

LEMMA 1. Let X be a quasicomplete lcHs and let $u : C_o(T) \to X$ be a non weakly compact continuous linear map. Then $C_o(T)$ contains a subspace Y isometrically isomorphic to c_o. Moreover, there exists an equicontinuous set A in X^* such that $\Pi_{p_A} \circ u$ is a topological isomorhism of Y into $\widetilde{X_{p_A}}$.

Proof. Since u is not weakly compact, by Proposition 2 there exists an equicontinuous subset A of X^* such that u^*A is not relatively weakly compact in $M(T)$. By Proposition 1, u^*A is bounded in $M(T)$ and hence by Proposition 3 there exist a disjoint sequence $\{U_n\}_1^\infty$ of open sets in T and an $\epsilon > 0$ such that $\sup_{x^* \in A} |(x^* \circ m)(U_n)| > 2\epsilon$ for each n. Consequently, there exists $x_n^* \in A$ such that $|(x_n^* \circ m)(U_n)| > 2\epsilon$ for each n. Since $x_n^* \circ m$ is Borel regular by Proposition 5(i), there exists a compact $K_n \subset U_n$ such that $|(x_n^* \circ m)(K_n)| > 2\epsilon$.

Let $D(K_n) = \{U : U \text{ open }, K_n \subset U \subset U_n\}$ and for $U, V \in D(K_n)$, let $U \geq V$ if $U \subset V$. Then by the Borel outer regularity of $x_n^* \circ m$ in K_n, there exists $U_o \in D(K_n)$ such that $|(x_n^* \circ m)(U \backslash K_n)| < \epsilon$ for all $U \geq U_o$, $U \in D(K_n)$. Then by Theorem 50.D of Halmos [5], we can choose an open Baire set V_n such that $K_n \subset V_n \subset U_o$ so that $|(x_n^* \circ m)(V_n \backslash K_n)| < \epsilon$. Consequently, $|(x_n^* \circ m)(V_n)| > |(x_n^* \circ m)(K_n)| - |(x_n^* \circ m)(V_n \backslash K_n)| > \epsilon$. Thus $|(x_n^* \circ m)(V_n)| > \epsilon$ for each n.

Claim 1. Suppose V is an open Baire set in T with $|(x^* \circ m)(V)| > \epsilon$. Then there exists $f \in C_o(T)$ such that $0 \leq f \leq \chi_V$, $||f||_T = 1$ and $|\int_T f d(x^* \circ m)| > \epsilon$. Consequently, there exist $(f_n)_1^\infty \subset C_o(T)$ with $||f_n||_T = 1$, $0 \leq f_n \leq \chi_{V_n}$ and $|\int_T f_n d(x_n^* \circ m)| > \epsilon$ for all n.

In fact, by § 14 of Dinculeanu [2], V is a countable union of compact G_δs and hence, there exists $\{C_k\}_{k=1}^\infty$ of compact G_δs such that $C_k \nearrow V$. Now by Urysohn's lemma there exists $h_k \in C_o(T)$ such that $0 \leq h_k \leq \chi_V$ with $h_k(t) = 1$ for all $t \in C_k$. Let $g_p = \max_{1 \leq k \leq p} h_k$. Then $\{g_p\}_{p=1}^\infty \subset C_o(T)$ and $g_p \nearrow \chi_V$. Then by the Lebesgue dominated convergence theorem there exists $p_o \in \mathbb{N}$ such that $|\int_T g_{p_o} d(x^* \circ m)| > \epsilon$. Taking $f = g_{p_o}$, the first part of the claim is established. The second part is immeditate from the first as $|(x_n^* \circ m)(V_n)| > \epsilon$ for all n.

By Proposition 1, $(x_n^* \circ m)_{n=1}^\infty$ is uniformly bounded in $M(T)$. Then by Rosenthal's lemma (see p.18 of [1], which holds for complex measures too) we can assume that the sequences (x_n^*) and (V_n) have been chosen such that

$$|(x_n^* \circ m)(V_n)| > \epsilon \quad \text{and} \quad |x_n^* \circ m|(\bigcup_{p \neq n} V_p) < \frac{\epsilon}{2} \quad (1)$$

for all n. Then by Claim 1 and Proposition 5(iii) we have

$$|x_n^* u f_n| = |\int_T f_n d(x_n^* \circ m)| > \epsilon \quad (2)$$

for all n. Moreover, $||f_n||_T = 1$ and $\text{supp } f_n \subset V_n$ for all n.

Let $Y = \{\sum_{n=1}^\infty \alpha_n f_n : (\alpha_n)_{n=1}^\infty \in c_o\}$. As $||f_n||_T = 1$ for all n and as (f_n) have disjoint supports, Y is isometrically isomorphic with c_o. Moreover, if $f = \sum_{n=1}^\infty \alpha_n f_n$ for some sequence $(\alpha_n)_{n=1}^\infty \in c_o$, then by (1) and (2) we have for each n

$$\begin{aligned} |x_n^* u(f)| &= |\int_T f d(x_n^* \circ m)| \\ &= |\alpha_n \int_T f_n d(x_n^* \circ m) + \int_{\bigcup_{p \neq n} V_p} f d(x_n^* \circ m)| \\ &\geq |\alpha_n|\epsilon - \int_{\bigcup_{p \neq n} V_p} |f| d(|x_n^* \circ m|) \end{aligned}$$

$$\begin{aligned} &\geq |\alpha_n|\epsilon - |x_n^* \circ m|(\bigcup_{p\neq n} V_p)||f||_T \\ &\geq |\alpha_n|\epsilon - \frac{\epsilon}{2}||f||_T. \end{aligned}$$

But $||f||_T = \sup_n |\alpha_n|$ and hence

$$\begin{aligned} ||(\Pi_{p_A} \circ u)(f)||_{p_A} = p_A(uf) &= \sup_{x^* \in A} |(x^*u)(f)| \\ &\geq \sup_n |(x_n^*u)(f)| \geq \epsilon||f||_T - (\frac{\epsilon}{2})||f||_T = (\frac{\epsilon}{2})||f||_T. \end{aligned}$$

Hence $(\Pi_{p_A} \circ u)|_Y$ is a topological isomprphism of Y with a subspace of $\widetilde{X}_{p_A}$ and this completes the proof of the lemma.

COROLLARY. *If X is a Banach space and $c_o \not\subset X$, then every continuous linear map $u : C_o(T) \to X$ is weakly compact.*

Remark 1. In the proof of Theorem VI.2.15 of [1] there is no hint as to the construction of the sequence (f_n) in $C(\Omega)$ with the desired properties. One has to invoke Theorem 50.D of [5] and the fact that every open Baire set is a countable union of compact G_δs. (See the proof of Claim 1 above.)

We need the following result from [9]. For the sake of completeness we include its proof.

LEMMA 2. $C_o(T)$ *has the strict Dunford-Pettis property (briefly, (SDP)-property). That is, for each weakly compact operator $u : C_o(T) \to X$, X a quasicomplete lcHs, u transforms each weak Cauchy sequence in $C_o(T)$ into a convergent sequence in X.*

Proof. If (f_n) is weakly Cauchy in $C_o(T)$, then it is a norm bounded sequence converging pointwise to some function f in T and clearly, f is also bounded and is Borel measurable. By Proposition 2 and by the fact that $m(E) = u^{**}(\chi_E)$ for $E \in \mathcal{B}(T)$ (see Proposition 5(v)), the representing measure m has range in X and consequently, by Proposition 5(i) and by the Orlicz-Pettis theorem for lcHs (see [6]) m is σ-additive in τ. Then by (vi) of Proposition 5 and (ii) of Proposition 4 we have

$$\lim_n uf_n = \lim_n \int_T f_n dm = \int_T f dm \in X.$$

Hence the result holds.

This completes the proof of the lemma.

From Lemmas 1 and 2 we shall now deduce the main theorem (which is the same as Theorem 12 of [9]).

THEOREM 1(Theorem 12 of [9]). *Let $u : C_o(T) \to X$ be a continuous linear map and let X be a quasicomplete lcHs. Then the following are equivalent:*

(i) u is uncoditionally convergent;

(ii) u is weakly compact.

(iii) u maps sequences that tend to zero weakly into sequences convergent to zero.

(iv) u maps weak Cauchy sequences into τ-Cauchy sequences.

(v) If (f_n) is a bounded sequence in $C_o(T)$ with $f_n \cdot f_l = 0$ for $n \neq l$, then $\lim_n u(f_n) = 0$.

Proof.

(i)⇒(ii) If $u : C_o(T) \to X$ is not weakly compact, then by Lemma 1 there exist an equicontinuous subset A of X^* and a subspace Y of $C_o(T)$ isometric with c_o such that $(\Pi_{p_A} \circ u)|_Y$ is a topological isomorphism with a subspace of $\widetilde{X}_{p_A}$. By (i) $\Pi_{p_A} \circ u$ maps weakly unconditionally Cauchy series in Y into unconditionally convergent series in $\widetilde{X}_A$. This is impossible as c_o contains plenty of nonconvergent weakly unconditionally Cauchy series. Hence u is weakly compact.

(ii) ⇒(i) and (iv) by Lemma 2.

(iv)⇒(iii) A sequence weakly convergent to zero in $C_o(T)$ is norm bounded and converges to zero pointwise. Then (iii) holds by Proposition 5(iii), the Lebesgue bounded convergence theorem, and the Hahn-Banach theorem.

(iii)⇒(v) Such a norm bounded sequence (f_n) converges to zero pointwise and hence by the Lebesgue bounded convergence theorem is weakly convergent to zero. Then by hypothesis (iii), $\lim_n u(f_n) = 0$ in τ.

(v)⇒(ii) If u is not weakly compact, then as in the proof of Lemma 1 we have an equicontinuous subset A of X^*, an $\epsilon > 0$, a sequence $(f_n)_1^\infty \subset C_o(T)$ with disjoint supports such that $||f_n||_T = 1$ for all n and a sequence $(x_n^*)_1^\infty$ in A with $|\int_T f_n d(x_n^* \circ m)| > \epsilon$ for all n. Then by Proposition 5(iii), $||u(f_n)||_{p_A} \geq |x_n^*(uf_n)| = |\int_T f_n d(x_n^* \circ m)| > \epsilon$ for all n. This contradicts (v) and hence u is weakly compact.

This completes the proof of the theorem.

Now we deduce the first part of Theorem 5.3 of Thomas [12] as a corollary of the above theorem.

COROLLARY (First part of Theorem 5.3 of [12]). *Let X be a quasicomplete lcHs with $c_o \not\subset X$. Then every continuous linear map $u : C_o(T) \to X$ is weakly compact. (Then by Propositions 2 and 5 and the Orlicz-Pettis theorem, the representing measure m of u has range in X and is σ-additive in τ.)*

Proof. Let $(f_n)_{n=1}^\infty$ be a sequence of functions in $C_o(T)$ such that $\sum_{n=1}^\infty |\int_T f_n d(\mu)| < \infty$ for each $\mu \in M(T)$. Let $u : C_o(T) \to X$ be a continuous linear map with the representing measure m.

Then by Proposition 5(i), $x^* \circ m \in M(T)$ for each $x^* \in X^*$ and hence by hypothesis on $(f_n)_{n=1}^{\infty}$ and by Proposition 5(iii) we have

$$\sum_{n=1}^{\infty} |x^*(uf_n)| = \sum_{n=1}^{\infty} | \int_T f_n d(x^* \circ m)| < \infty.$$

Since $c_o \not\subset X$, by Theorem 4 of Tumarkin [13] it follows that $\sum_{n=1}^{\infty} u(f_n)$ converges unconditionally in X (in τ). Thus u is an unconditionally convergent operator and hence by (i)$\Rightarrow$(ii) of Theorem 1, u is weakly compact.

Remark 2. The above corollary is deduced from Lemma 1 via Theorem 1, while its Banach space analogue is immediate from Lemma 1. Note that strict Dunford-Pettis property of $C_o(T)$ is not used in proving the corollary.

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