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A notion of manipulability based on lifting preferences

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Abstract

We define in a very precise and new manner the concept of manipulability of general (classical) social choice functions. In order to do that, we need to lift individual preferences over elements of a set of alternatives X to preferences over subsets of X. We establish a new theorem of impossibility of being free of manipulation à la Gibbard-Satterthwaite.

key words. Manipulation, Preferences, Merging, Social Choice, Qualitative Decision Making

Resumen

Definimos una nueva manera muy precisa el concepto general de manipulabilidad de funciones (clásicas) de elección social. Para hacer esto, necesitamos levantar preferencias individuales sobre elementos de un conjunto de alternativas X a preferencias sobre subconjuntos de X. Establecemos un nuevo teorema de la imposibilidad distinto a la noción de manipulabilidad establecida por Gibbard-Satterthwaite .

1 Introduction

Studying preferences is now a common issue from different domains: (qualitative) Decision Making under Uncertainty [10, 11], Merging Information in Logical frameworks [19], Knowledge Representation [8], Social Choice [1], etc. Via this common issue there are some interesting problems which can be translated from one domain to another. That is the case of recent works in Information Fusion and Belief Merging [15, 6]. In these works the concept of *strategy-proofness*, coming from Social Choice Theory, is studied. These analogies are very interesting and, in most cases, open new paths of research. They have also the virtue of bringing back the research to the domain of origin and there, addressing some important questions.

With the aim of having a better understanding of the problem of manipulability, we must go to the sources in Social Choice. Below, we give some intuitive explanations of concepts involved in this work. We hope that they will help to understand and clarify the panorama of key notions in the study of selection functions and manipulability. We also raise some problems and propose some solutions.

In order to explain the manipulability problem, let us begin by explaining very roughly what Social Choice is. When we face the problem of selecting the best candidates of a list, related to some individual preferences over the candidates, we are actually facing a problem of Social Choice. More precisely, the general issue addressed by Social Choice Theory is the study of such procedures of selection.

The first important question asked in this domain is what a good selection procedure is (they are called social choice functions) and, of course, if such functions exist. The main idea, in order to establish what a *good* social choice function is, consists in defining a set of rational properties that the functions have to satisfy. A very small set of these properties that seem very sensible (absence of dictatorship, Pareto dominance, transitive explanations, independence of irrelevant alternatives and the totality of the procedure -see Section 2 for a precise formulation-) appears as the minimum set of conditions that a good function has to satisfy. As a matter of fact, they are incompatible. Precisely, the surprising[1] result of Arrow [1, 17], known as Arrow's Impossibility Theorem, says that there are no such functions (in Section 2 we find the precise formulation).

Another important question can be formulated in the following terms. What are the properties of social choice functions that guarantee they are free of manipulation? Since the precise and partial formulation of this problem by Gibbard and Satterthwaite in 1973 [16, 26] and their interesting solution, relatively few works have been done in this domain. The Gibbard-Satterthwaite's Theorem states that a large class of social choice functions are manipulable. The class of functions where the result applies concerns the functions mapping a set of individual preferences into an alternative (and having a range of cardinal bigger or equal to three, *i.e.* having al least three outputs). Note that for this kind of functions it is quite natural and easy to define manipulability: a function f is manipulable if there exists an input u (thought as the vector of the true preferences[2] of individuals), there exists an individual i such that its true preference is $\preceq_i$ and another preference $\preceq'_i$ such that if u' is the input resulting of replacing in u, the preference $\preceq_i$ by $\preceq'_i$, we have $f(u') \prec_i f(u)$. This can be interpreted as follows: in situation u, it is most convenient for individual i (the manipulator) to lie, *i.e.* giving $\preceq'_i$ as its preferences, than to give its true preferences! Doing that, the result obtained $f(u')$ is strictly preferred by him than $f(u)$.

[1]There are some controversial opinions about the "surprising" character of Arrow's Theorem, see for instance [32].

[2]In general, in this paper the preference relations are denoted by the symbol $\preceq$; the strict relation associated to it will be denoted by $\prec$. The expressions $x \preceq y$ and $x \prec y$ mean x is not less preferred than y and x is strictly preferred to y, respectively (see Section 2 fore more details).

We have to mention the works of Dugann and Schwartz [14], Barberà et al. [2], Benoit [5] where they give very general manipulability theorems for some classes of social choice functions. The classes concern functions giving sometimes ties as a result. They consider only restricted domains and the functions they consider are in fact *social choice correspondences* which are not exactly social choice functions. See Section 5 for more details.

Other results which are very interesting are those of Conitzer et al. [7] concerning the complexity of calculating the *lie* that the *manipulator* has to tell in order to obtain a better result.

Having explained a little bit about the problem of manipulation in Social Choice Theory, let us say a few more words about the motivations of this work. The problem of aggregation of preferences can be translated to frameworks of negotiation processes [22, 23, 15]. In those frameworks it is important to have procedures free of manipulation. Thus, the study of manipulation in a general and abstract framework benefits and brings together communities as those of Artificial Intelligence and Economy, studying particularly mechanisms of merging information and social choice theory respectively. Thus, our first goal is to have a better understanding of the manipulation problem. What has been done? What remains to be done? We claim that manipulation, in the most general framework of selection functions, remains to be studied and we will explain why. Our second goal is to set some problems derived from definitions of manipulability in more general settings. Our third goal is to give a (weak) definition of manipulability in the general case in a very natural way. In order to do that we need to lift a preference over alternatives to a preference over sets of alternatives (the *ties*) which can be the outcome in the general case. This is necessary because, in order to define manipulation, we need to express when a set of alternatives is preferred over another one.

We organize the rest of this work as follows. In the next Section we present the basic concepts and classical results we need in the subsequent Sections. Section 3 presents our main tool, the concept of *lifting*. Therein we define four natural liftings. In Section 4 we give the concept of manipulability related to a lifting and examine the manipulability with respect to some of liftings previously defined. Section 5 is dedicated to compare our work with other related results. Finally, we end with a Section containing some remarks about our results and we present some lines of future work.

2 Preliminaries

We suppose we have a nonempty and finite set N of *individuals*. Let n be the cardinality[3] of N, actually we suppose $N = \{1, \dots, n\}$. Let X be a nonempty finite set. X will be called the set of *alternatives*. An (individual) *preference* will be a total pre-order over X, *i.e.* a transitive and total relation. Note that reflexivity of $\preceq$ follows of totality. $x \preceq y$ is read x is not less preferred than y. Note that economists denote this relation the other way around, *i.e.* for them, $x \succeq y$ means x is not less preferred than y. In any case, the best elements are in the left of the relation.

The relation of strict preference associated to a preference $\preceq$ is denoted $\prec$ and is defined by $x \prec y$ iff $x \preceq y$ and $y \not\preceq x$. When $x \prec y$ we read x is preferred to y. Note that, if the $\preceq$ is a total pre-order the relation associated to it, $\prec$ is a weak order (or equivalently a modular relation[4]), *i.e.* an asymmetric and negatively transitive relation[5].

The set of total pre-orders over X will be denoted P. An element u of P^n (the cartesian product of P, n times) is called a *profile*. In the profile $u = (\preceq_1, \dots, \preceq_n)$, the preference $\preceq_i$ denotes the preference of the individual i. A nonempty set of X is called an *agenda* (the names profile and agenda are the technical terms used by economists in Social Choice Theory). The set of agendas will be denoted $^*(X)$.

If V is an agenda and $\preceq$ is a total pre-order over X we define the set of minimal elements of V with respect to $\preceq$, denoted $\text{mín}(V, \preceq)$ as follows:

$$\text{mín}(V, \preceq) = \{x \in V : \forall y (y \prec x \Rightarrow y \notin V)\}$$

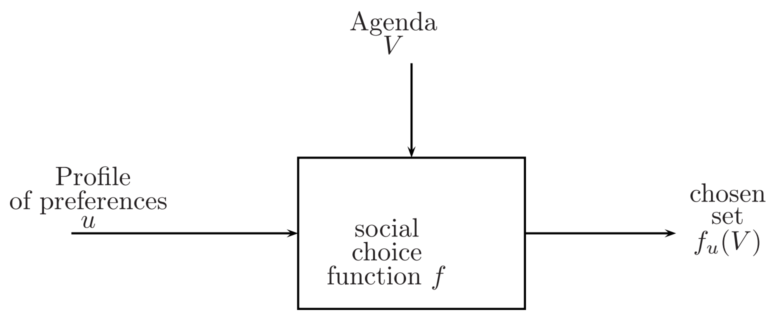
Definition 1. *A* social choice function *is a function* $f : P^n \times \mathcal{P}^*(X) \longrightarrow \mathcal{P}^*(X)$ *such that* $f(u, V) \subseteq V$. *Often* $f(u, V)$ *will be denoted* $f_u(V)$.

Note that in Social Choice Theory (see for instance [17]), they have a "curryfied" vision of what I call a social choice function. First they consider a function from profiles into functions mapping agendas into agendas. These functions are called social choice rules and the functions C mapping agendas into agendas with the property that $C(v) \subseteq V$ are called choice functions. The following two diagrams illustrate the different points of view.

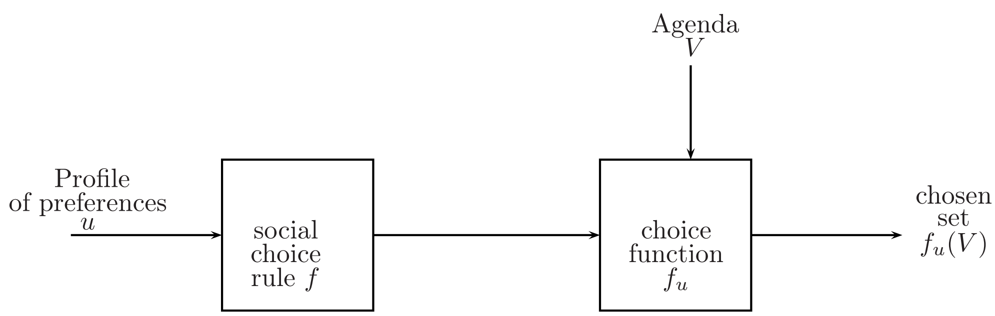
[3] As usual the cardinality of a set A will be denoted $|A|$.

[4] A relation R over X is said to be modular iff there exist a linear order $(\Omega, <)$ and a function $f : X \longrightarrow \Omega$ such that $xRy \Leftrightarrow f(x) < f(y)$, for every $x, y \in X$.

[5] A relation R over X is asymmetric iff $xRy \Rightarrow \neg(yRx)$, for every $x, y \in X$. A relation R over X is negatively transitive iff $\neg(xRy) \wedge \neg(yRz) \Rightarrow \neg(xRz)$, for every $x, y, z \in X$.



A social choice function (the view of definition 1)



A social choice rule plus choice function (the economists' view)

Now we define some rational desirable properties for social choice functions. The fact that f is total, $|N| \geq 3$ and $|X| \geq 3$ is known as the *Domain Standard Condition*. The totality of the function is desirable because we want to have a procedure that gives a result in any given situation.

Let V be a nonempty subset of X. Let $\preceq$ a preference. We denote by $\preceq\restriction_V$ the restriction to A of the relation $\preceq$. If $u = (\preceq_1, \ldots, \preceq_n)$ then $u \restriction_V = (\preceq_1\restriction_V, \ldots, \preceq_n\restriction_V)$. A social choice function f satisfies the *Independence of Irrelevant Alternatives* Property if and only if for all $V \in^* (X)$ and for all $u, u' \in P$ if $u \restriction_V = u' \restriction_V$ then $f_u(V) = f_{u'}(V)$. This condition state that the result of selecting on an agenda V depends only on the individual preferences on V.

We say that a social choice function f satisfies the *Strong Pareto Condition* if for all $u = (\preceq_1, \cdots, \preceq_n)$ and V the following holds: if for any $i \in N$, $x_i y$, there exists $j \in N$, $x \prec_j y$ and $x \in V$, then $y \notin f_u(V)$. In particular, if f satisfies the Domain Standard Condition and $V = \{x, y\}$, the Strong Pareto Condition says that if for all the individuals x is not less preferred than y and if for at least one individual x is preferred to y, then selecting the best elements of V, will give only x.

A social choice function f satisfies *Transitive Explanations* if for all profile u there exists a total pre-order $_u$ such that $f_u(V) = \text{mín}(V,_u)$, for any agenda V. This is a very interesting property. It says that there is a very uniform way for choosing the best elements of agendas when the profile is fixed. In other words, in the economists view, the social choice rule (the first step in the process) consists in giving an aggregation total pre-order $\preceq_u$ to the input u and then the choice function (the second step) consists in taking the minimal elements (the preferred ones) of the agenda V with respect to this relation $\preceq_u$.

All the four previous properties seem to be very good and rational properties for a social choice function. A last property which is desirable for a social choice function is the *absence of a dictator* where a dictator is defined in the following way: the individual i is a *dictator* for f if for all $u = (\preceq_1, \ldots, \preceq_n)$ in P^n, for all $V \in^* (X)$, and for all $x, y \in X$, if $x \prec_i y$ and $x \in V$ then $y \notin f_u(V)$. It is interesting to note that if there is a dictator i, and if in the profile u the preference $\preceq_i$ is a linear order[6] then $f_u(V)$ is the preferred element (the minimum) of V with respect to $\preceq_i$. Another interesting remark is that, in presence of Transitive Explanations and the j-indiference[7], the preference aggregation relation $\preceq_u$ coincides with the preference relation of the dictator, that is, the social choice rule is a projection function (giving the j-th component, where j is the dictator).

Now, having stated the previous properties we can formulate the notable Arrow's result [1]. It tells us that it is impossible to have a function for which these five good properties hold (for a proof we can also see [17] or [21]; in the last reference one can find a very interesting analysis of the proof). More precisely, it can be stated as follows.

Theorem 2. *If a social choice function f satisfies the Domain Standard Condition, the Independence of Irrelevant Alternatives Property, the Strong Pareto Condition and Transitive Explanations, then f has a dictator.*

Based on Arrow's Theorem, Gibbard [16] and Satterthwaite [26] independently give a proof of what is today known as the Gibbard-Satterthwaite's Theorem. In order to settle this result we need the following concepts.

Definition 3. *A function $g : P^n \longrightarrow X$ will be called a* Voting scheme.

If a Voting scheme g is onto[8], $|X| \geq 3$ and $n \geq 3$, we will say that g satisfies the *Gibbard*

[6] A linear order is a transitive, antisymmetric and total relation.

[7] That is is $x \sim_j y$ then $x \sim_u y$

[8] Actually the Gibbard's condition is much weaker than onto, he only asks that the range of g has at least three elements.

Standard Domain Condition.

If u is a profile and $\preceq$ is a preference we denote $u[\preceq /i]$ the profile that coincides with u for the individuals $j \neq i$ and for i is $\preceq$, that is to say if $u = (\preceq_1, \ldots, \preceq_i, \ldots, \preceq_n)$, then $u[\preceq /i] = (\preceq_1, \ldots, \preceq, \ldots, \preceq_n)$.

The individual i is called a *Gibbard Dictator* for g if for all x there exists $\preceq^x$ such that for all profile u, $g(u[\preceq^x /i]) = x$. That is to say, if individual i wants x be the winner, he can obtain it by choosing well her preferences independently of the other individual preferences.

Definition 4. *A voting scheme g is said to be* manipulable *iff there exist k, $u = (\preceq_1, \ldots, \preceq_n)$ and $\preceq$ such that $g(u[\preceq /k]) \prec_k g(u)$.*

Note that this definition formalizes the concept of manipulation for voting schemes described in the Introduction. In other words, it says that the individual k (the manipulator) obtains a strictly better result with respect to $\preceq_k$ (his true preference) if he lies, that is, if he changes his preference to $\preceq$.

When a voting scheme g is manipulable and a triple $k, u, \preceq$ is a witness of the manipulability of g as in the previous definition, we say that such a triple is a *situation of manipulation.*

Theorem 5. *Any voting scheme g satisfying the Gibbard Standard Domain condition is manipulable or has a Gibbard Dictator.*

For a proof of this result one can see, of course, the sources [16, 26]. We can also find an interesting proof in [24]

It is quite clear that a social choice function (Definition 1) and a voting scheme (Definition 3) are very different. Actually, one could see a voting scheme g as generated by a very particular social choice function f, by the following statement: $g(u) = x$ iff $f(u, X) = \{x\}$. Of course, the class of social choice functions generating voting schemes via the previous statement is very restricted because this imposes the absence of ties when the agenda is the whole X.

Thus, a generalization of Theorem 5 to social choice functions is not so straightforward. Then, the first thing to do is to give a definition of manipulability for social choice functions, *i.e.* to find the definition corresponding to the Definition 3, in the setting of social choice functions. Some work has been done in the past five years in this direction. In Section 5 we will show the connections of that work to our work. In particular, we show some similarities and some differences.

One way to perform this, is considering only the functions satisfying Transitive Explanations and seeing the social choice function as a function taking values in P, *i.e.* the outputs are preferences. This can be done because the social choice functions having Transitive Explanations satisfy the following proposition.

Proposition 6. *Suppose that the social choice function f satisfies Transitive Explanations and Domain Standard condition. For each profile u define a total pre-order $_u$ as follows: $x_u y \Longleftrightarrow x \in f_u(\{x, y\})$. Then $_u$ it is the unique total pre-order that satisfies $f_u(V) = \text{mín}(V,_u)$.*

By the previous result, it is easy to see a function having Transitive Explanations $f : P^n \times^* (X) \longrightarrow^* (X)$ as a function $\hat{f} : P^n \longrightarrow P$. The function $\hat{f}$ is defined by $u \mapsto \preceq_u$ where $\preceq_u$ is the unique total pre-order satisfying $f_u(V) = \text{mín}(V,_u)$. Conversely having $\hat{f}$, mapping $u \mapsto \preceq_u$, we define f by putting $f(u, V) = \text{mín}(V,_u)$. By abuse of notation, we identify f and $\hat{f}$. With this identification in mind, we want to find out what a manipulation situation is. The obvious choice is to take a triple $k, u, \preceq$ such that $f(u[\preceq /k])$ is *strictly better* than $f(u)$. But the problem is that we need relations between preferences in order to give some sense to the previous phrase. Actually, we would need a preference $\sqsubseteq^{\preceq_k}$ over preferences such that $\preceq \sqsubseteq^{\preceq_k} \preceq'$ expresses that the preference $\preceq$ is better, relative to $\preceq_k$, than $\preceq'$. Although this kind of approach seems interesting and promising, it is not used in this work because of the difficulty to define a rational relation $\sqsubseteq^{\preceq_k}$. The relation built using the Kemeny distance is not very convincing.

The approach to tackle the problem of manipulability in this work is to take into account all the inputs of a social choice function. In particular, a situation of manipulability will be a quadruplet $k, u, \preceq, V$ where $k, u, \preceq$ are as before and V is an agenda such that $f_{u[\preceq/k]}(V)$ is better than $f_u(V)$, relative to the lifting of $\preceq_k$. To define this in a precise manner is the goal of the following Section.

3 Lifting preferences

Transferring the information from preferences over points into preferences over sets of points in a rational manner is an old task. It started many years ago. Perhaps the most common way, in the finite case (when X is finite), is through a probability p defined on X, which extends additively to subsets of X. Thus, one can define what is called a likelihood probabilistic relation over subsets of (events of) X: we put $E_1 \sqsubseteq E_2$ iff $p(E_1) \geq p(E_2)$.

We are considering in this section, in the first place, a way to transfer qualitative information from preferences over points into preferences over sets of points in a very natural manner that goes back to Shackle [27, 28] (and has been proposed in various formats by Lewis, Zadeh, Dubois, Spohn, Halpern, etc.). This is called a *comparative possibility measure* and corresponds to our definition of lifting I below.

Let us now turn to the lifting notion:

Definition 7. *A map $\preceq \mapsto \sqsubseteq_{\preceq}$ that sends a preference over X, $\preceq$, into a preference over ${}^*(X)$, $\sqsubseteq_{\preceq}$, is called a lifting iff the following condition holds for any pair, $x, y \in X$:*

$$x \preceq y \iff \{x\} \sqsubseteq_{\preceq} \{y\}$$

Thus if $\preceq \mapsto \sqsubseteq_{\preceq}$ is a lifting, the total pre-order $\sqsubseteq_{\preceq}$ is an "extension.ºf the total pre-order $\preceq$.

Definition 8. *Let be a total pre-order over X. Let A and B any elements of ${}^*(X)$. We define:*

$$A^I B \iff \exists a \in \text{mín}(A,) \wedge \exists b \in \text{mín}(B,) \;\; s.t. \;\; ab$$

$$A^{II} B \iff A^I B \;\; or \;\; (A \simeq^I_B \;\; \& \; |A| \leq |B|)$$

$$A^{III} B \iff A^I B \;\; or \;\; (A \simeq^I_B \;\; \& \; |\text{mín}(A, \preceq)| \leq |\text{mín}(B, \preceq)|)$$

$$A^{IV} B \iff A^{III} B \;\; or \;\; (A \simeq^{III}_B \;\; \& \; |A| \leq |B|)$$

The relation ^{I} associated to a , as we said previously, is in fact the comparative possibility relation associated with the "possibility measure" (see for instance [12, 13]). It extends in a natural way the preferences over elements of X expressed by , to preferences over ${}^*(X)$ expressed by ^{I}. The meaning of $A^I B$ can be stated as follows: the best elements of A (related to) are preferred or indifferent to the best elements of B (related to); or, even more graphically, that the best elements of A are in a lower level or in the same level than the best elements of B.

The relation ^{II} can be viewed (in a quantitative way) as a refinement of the first one with a uniform probability. It can be explained in words in the following manner: AB if and only if, the best elements of A (related to) are strictly preferred to the best elements of B (relative to) or in the case the best elements of A and B (related to) are in the same level, A has less or equal elements than B, *i.e.* when the best of A and B are in the same level, one prefers the most precise set (the smaller one).

The relation III is, actually, the leximin-refined possibility relation. See for instance [9].

The relation IV is, as in the case of the second relation, a refinement of the third one by a uniform probability.

In order to illustrate Definition 8, take the following graphical example. Here, the pre-order $\preceq$ is represented by levels and the elements of lowest levels are better than elements in highest levels.

Example 9.

In the figure (a) is represented by levels. Here, we can observe that A and B satisfy $A^I_{\prec}B$, *therefore* $A^{II}B$, $A^{III}B$ *and* $A^{IV}B$.

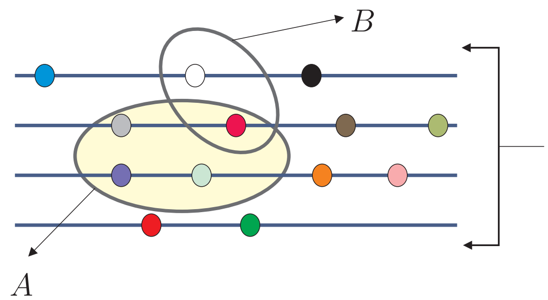


Figure (a)

In the figure (b) the sets C and D satisfy the following relations: $C \approx^I D$, $|C| = |D|$, *therefore* $C \approx^{II} D$ *but* $|min(D)| < |min(C)|$, *so* $D \sqsubset^{III}_{\preceq} C$ *and* $D \sqsubset^{IV}_{\preceq} C$.

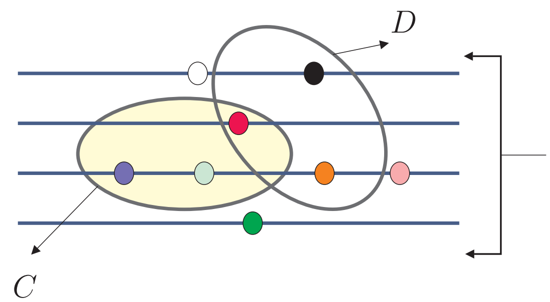


Figure (b)

In the figure (c) the sets E and F satisfy the following relations: $E \approx^I F$ *but* $|E| < |F|$, *therefore* $E^{II}F$. *As* $|min(E)| = |min(F)|$, $E \approx^{III} F$ *but* $E^{IV}F$

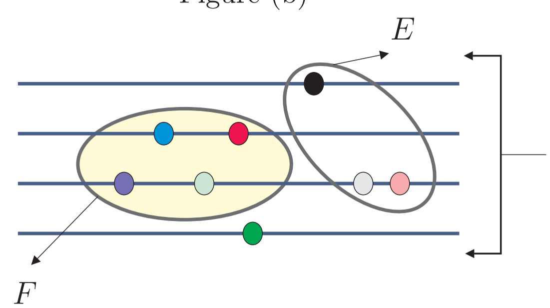


Figure (c)

Proposition 10. *The four mappings* $\preceq \mapsto \sqsubseteq^{\alpha}_{\preceq}$, *for* $\alpha \in \{I, II, III, IV\}$ *are indeed liftings.*

Proof: The verification of the condition in Definition 7 is trivial. The point to check is that for every $\alpha \in \{I, II, III, IV\}$, the relation $\sqsubseteq^{\alpha}_{\preceq}$ is a total pre-order. For $\alpha = I$ it is straightforward.

For the rest of α it is enough to note that the lexicographical relation associated to two total pre-orders is a total pre-order.

Observation 11. *We can imagine other* natural *ways to define liftings. What is the best way, if any? That is an open problem. In the sequel we divide the liftings into two categories: those for which the extension of manipulability theorem holds and the others. This is the beginning of a classification.*

4 A manipulability theorem

One of the main contributions of this work is the following simple definition:

Definition 12. *Let* $f : P^n \times \mathcal{P}^*(X) \longrightarrow \mathcal{P}^*(X)$ *be a social choice function.* f *is said to be manipulable (related to a lifting* $\preceq \mapsto \sqsubseteq_{\preceq}$*) iff*

there exist k*, , *u*, and* V *such that*

$$f_{u[/k]}(V)_{k} f_u(V).$$

Definition 13. *A social choice function* f *satisfies the* Strong Standard Domain Condition *(SSD) if it satisfies the Standard Domain Condition and for all* $x \in X$ *there exists a profile* u *such that for all* y*,* $f_u(\{x, y\}) = \{x\}$.

This condition means that for any candidate x there is a profile u such that for all binary agenda V (that is $|V| = 2$) containing x the result is x, in other words, u makes x winner against any other candidate.

It is interesting to note that the Pareto Condition and the Standard Domain condition imply together the Strong Standard Domain Condition. To see that, it is enough to take as the profile u a profile in which each individual has the alternative x as the most preferred one. Thus, for any individual i and any other alternative y, $x \prec_i y$. Then, by Pareto condition and $y \notin f_u(\{x, y\})$ and, necessarily, by Standard Domain condition, $f_u(\{x, y\}) = \{x\}$.

Definition 14 (Weak Dictator (WD)). *A social choice function* f *has a weak dictator* k *if for all* $x \in X$*, there exists* ^{x} *such that for all* $y \in X$*,* $x \in f_{u[^x/k]}(\{x, y\})$.

In contrast with the notion of dictator, which is an excluding notion, the notion of weak dictator is an including one.

The following lemma will be useful in the proof of Theorem 16.

Lemma 15. *Let f be a social choice function satisfying Transitive Explanations and SSD. Then for any x, there exists a profile u such that $f_u(X) = \{x\}$.*

Proof: For a given x, take u such that for all y we have $f_u(\{x, y\}) = \{x\}$ (the existence of u is guaranteed by SSD). Since f satisfies Transitive Explanations, for any agenda $V \in^* (X)$, $f_u(V)$ is determined by $_u$ in the following way (see Proposition 6):

$$f_u(V) = \text{mín}(V,_u)$$

We claim that, by the choice of u, $\text{mín}(X,_u) = \{x\}$. Towards a contradiction, suppose this is not the case; therefore, there exists y such that $y_u x$; thus $\text{mín}(\{x, y\},_u) \neq \{x\}$, i.e. $f_u(\{x, y\}) \neq \{x\}$, a contradiction.

Theorem 16. *Let $f : P^n \times \mathcal{P}^*(X) \longrightarrow \mathcal{P}^*(X)$ be a social choice function satisfying the Strong Standard Domain (SSD) and Transitive Explanations (TE).*
Then, related to the liftings $\preceq \mapsto \sqsubseteq^{II}_{\preceq}$ and $\preceq \mapsto \sqsubseteq^{IV}_{\preceq}$, f is manipulable or f has a weak dictator.

Proof: We make the proof for the lifting $\preceq \mapsto \sqsubseteq^{II}_{\preceq}$. For the lifting $\preceq \mapsto \sqsubseteq^{IV}_{\preceq}$ the proof is similar. Let $f : P^n \times \mathcal{P}^*(X) \longrightarrow \mathcal{P}^*(X)$ such that f satisfies SSD. Let $\leq^*$ be a linear order over X fixed for the rest of the proof.

Define $g :^n \longrightarrow X$ by putting

$$g(u) = \text{mín}(f_u(X), \leq^*). \tag{1}$$

It is clear that g is a voting scheme. From Lemma 15 follows easily that g satisfies the Gibbard Standard Domain Condition. Thus, by Theorem 5, g has a Gibbard Dictator or g is manipulable.

Now, in order to finish the proof it is enough to show that the following claims hold:

Claim 17. *If g has a Gibbard Dictator, then f has a Weak Dictator.*

Proof: By the hypothesis there exists $k \in N$ such that for any $x \in X$, there exists $_x \in$ such that for any $u \in^n$ we have

$$g(u[_x/k]) = x,$$

that is to say, $x = \text{mín}(f_{u[_x/k]}(X), \leq^*)$. Thus, $x \in \text{mín}(X, \preceq_{u[_x/k]})$. Necessarily, $x \in \text{mín}(\{x, y\}, \preceq_{u[_x/k]}$), that is $x \in f_{u[_x/k]}(\{x, y\})$. Therefore k is also a Weak Dictator for f.

Claim 18. *If g is manipulable then f is manipulable.*

Proof: Assume g is manipulable. Then there exists a manipulation situation $u \in^n$, $k \in N$, and $\in$ such that

$$g(u[/k]) \prec_k g(u). \tag{2}$$

Define x and y by the following two equations: $g(u[/k]) = \{x\}$ and $g(u) = \{y\}$. It will be enough to verify the following statement

$$f_{u[/k]}(\{x,y\})^{II}_{k} f_u(\{x,y\}). \tag{3}$$

There are four possible cases according to the image of $f_{u[/k]}(\{x,y\})$ and $f_u(\{x,y\})$:

(a) $f_{u[/k]}(\{x,y\}) = \{x\}$

(b) $f_{u[/k]}(\{x,y\}) = \{x,y\} \wedge x <^* y$

(c) $f_u(\{x,y\}) = \{y\}$

(d) $f_u(\{x,y\}) = \{x,y\} \wedge y <^* x$

The case (b) & (d) is impossible. The rest of the cases, *i.e.* (a) & (c), (a) & (d) and (b) & (c) are possible. Let us examine the case (a) & (d). To see that the statement 3 is true, it is enough to verify $\{x\} \sqsubset^{II}_{\preceq_k} \{x,y\}$. By definition of x and y and the statement 2, $x \prec_k y$. Thus, the minimal elements with respect to $\preceq_k$ of $\{x\}$ and $\{x,y\}$ are in the same level. But $|\{x\}| < |\{x,y\}|$. Therefore, $\{x\} \sqsubset^{II}_{\preceq_k} \{x,y\}$.

Now, let us examine the case (a) & (c). Again, by definition of x and y and the statement 2, $x \prec_k y$. Thus, $\{x\} \sqsubset^{II}_{\preceq_k} \{y\}$.

For the case (b) & (c), we have, by definition of x and y and the statement 2, $x \prec_k y$. Thus, $\{x,y\} \sqsubset^{II}_{\preceq_k} \{y\}$.

We have shown that f is manipulable.

Let us finish this Section with an example illustrating some of the previous results.

Example 19. *Let* $X = \{x,y,z\}$ *and* $N = \{1,2,3\}$. *We define a social choice function* $f : P^3 \times^* (X) \longrightarrow^* (X)$, *the Borda rule, as follows. First, for each preference* $\preceq$ *and any* $\alpha \in X$ *we define the Borda rank of* α *relative to* $\preceq$, *denoted* $r_{\preceq}(\alpha)$, *as the level in which* α *appears in the pre-order* $\preceq$. *For instance, if* $x \prec y \prec z$ *we have* $r_{\preceq}(x) = 0$, $r_{\preceq}(y) = 1$ *and* $r_{\preceq}(z) = 2$. *We extend additively this notion to profiles. More precisely, if the profile* u *is* $(\preceq_1, \preceq_2, \preceq_3)$, *we define* $r_u(\alpha) = \Sigma^3_{i=1} r_{\preceq_i}(\alpha)$. *For instance, if*

$$u = \left\{ \begin{array}{ccc} & x & y \\ z & y & z \\ xy & z & x \end{array} \right\}$$

then $r_u(x) = 2$, $r_u(y) = 3$ *and* $r(z) = 2$.

We can associate to a profile u*, a preference* $\preceq_u$ *by putting* $\alpha \preceq_u \beta$ *iff* $r_u(\alpha) \leq r_u(\beta)$*. It is very easy to prove that* $\preceq_u$ *is a preference. Finally we put* $f_u(V) = \text{mín}(V, \preceq_u)$*. Thus, for instance, for the previously* u *defined we have*

$$\preceq_u = \left\{ \begin{array}{c} \\ y \\ xz \end{array} \right\}$$

and $f_u(X) = \{x, z\}$*. So, this function is not generating a voting scheme* g *via the equation* $g(u) = f_u(X)$*. Nevertheless, this function satisfies four of the five properties of Arrow's Theorem (Theorem 2). The only property which does not hold is the Independence of Irrelevant Alternatives.*

Now if you are calling a wise man *to solve conflicts you can have a voting scheme. In order to do that, fix a linear order (the wise man)* $\leq^*$ *over* X *and put* $g(u) = \text{mín}(f_u(X), \leq^*)$*. It is not hard to see that this* g *is a voting scheme satisfying the Gibbard Standard Condition and* g *does not have a weak dictator, so by virtue of Gibbard-Satterthwaite's Theorem (Theorem 5),* g *is manipulable. Actually, if* $z <^* y <^* x$ *and* u *is defined as before, we have* $g(u) = \text{mín}(f_u(X), \leq^*)$*,* i.e. $g(u) = \text{mín}(\{x, z\}, \leq^*) = z$ *and if*

$$u' = \left\{ \begin{array}{ccc} z & x & y \\ y & y & z \\ x & z & x \end{array} \right\}$$

then $f_{u'}(\{x, y, z\}) = \{x\}$*, so* $g(u') = x$*. But if we denote by* $\preceq'$ *the relation satisfying* $x \preceq' y \preceq' y \preceq' z$*,* u' *is in fact* $u[\preceq' /1]$*. But remember that in the true preferences of individual 1 (the first projection of* u*) we have* $x \prec_1 z$*, so*

$$g(u[\preceq' /1]) = g(u') = x \prec_1 z = g(u)$$

that is, the triple 1*,* u *and* $\preceq'$ *is a situation of manipulation for* g*. Thus, the individual 1, lying, obtains a result which he really prefers.*

This example can have a funny interpretation: suppose that N is a set of three expert referees evaluating a paper for a conference; x is acceptation, y is revision and z is rejection. If the profile of preferences about the paper is expressed by u and the *wise man* is the following strict editorial policy of the Committee of Program: the first choice is rejection, the second choice is revision and the least preferred option is acceptation. The paper will be rejected if the procedure adopted is g; but if the first referee considers that the paper has to be accepted, he can change his preferences to $\preceq'$ and then the review result will change favorably to him: the paper will be accepted.

Note that in this case almost the same data illustrates Theorem 16, that is, $1, u, \preceq', \{x, z\}$ is a situation of manipulation for f with respect to the lifting II, because

$$f_{u[\preceq'/1]}(\{x,z\}) = \{x\} \sqsubset^{II}_{\preceq_1} = \{x,z\} = f_u(\{x,z\})$$

5 Related works

For a very complete survey about manipulability one can see the recent book of Alan D. Taylor [31]

In the work of Duggan and Schwartz [14] we can find a very general notion of manipulability. For a detailed exposition we can see [25]. We resume here the main concepts. First let us define the concept of *social choice correspondence*. We denote by L the set of linear orders over X. Note that L is a proper subset of P, the total pre-orders. A function $f : L^n \longrightarrow \mathcal{P}^*(X)$ is called a social choice correspondence. Note the difference between this notion and the notion of social choice function which considers another parameter, the agendas. Furthermore, the domain of profiles is very restrictive: individuals are allowed to have only strict preferences between alternatives.

A *probability assessment* over V is a function $\lambda : V \longrightarrow (0,1]$ such that $\sum_{v \in V} \lambda(v) = 1$. A *utility* function ν is a function $\nu : X \longrightarrow \mathbb{R}$. A utility function ν represents a preference $\prec$ iff for any $x, y \in X$, $x \prec y$ iff $\nu(x) > \nu(y)$.

Let $\sqsubseteq$ be a total pre-order over $\mathcal{P}^*(X)$. Let $\preceq \in L$; $\sqsubseteq$ is said to be $\preceq$-consistent iff $\sqsubseteq$ restricted to singletons is a linear order and for each $V, V' \in \mathcal{P}^*(X)$, $V \sqsubset V'$ implies that for each pair of probability assessments over V and V', there is a utility function ν representing $\preceq$ such that

$$\sum_{x \in V} \lambda_V(x)\nu(x) > \sum_{x \in V'} \lambda_{V'}(x)\nu(x)$$

Note that if $\sqsubseteq$ is $\preceq$-consistent then $\sqsubseteq$ is an extension of $\preceq$, *i.e.* $x \prec y$ implies $\{x\} \sqsubset \{y\}$.

A social choice correspondence f is *manipulable* if there exists an individual i, $u \in L^n$, $\preceq \in L$ and a $\preceq_i$-consistent preference $\sqsubseteq_i$ such that

$$f(u[\preceq/i]) \sqsubset_i f(u)$$

In this notion of manipulability the concept of $\preceq_i$-consistent plays the role of lifting. But this is evidently a numerical way for lifting preferences. This contrasts with the purely combinatorial notion characterizing our liftings. Actually, it is easy to see that there is a total pre-order $\preceq$ such that $\sqsubseteq^{II}_{\preceq}$ is not $\preceq$-consistent. Take for instance $\preceq$ such that there are x, y with $x \preceq y$ and $y \preceq x$

(*i.e.* they are equally preferred). Define $V = \{x\}$ and $V' = \{x, y\}$. Then it is clear that $V \sqsubset^{II}_{\preceq} V'$. Towards a contradiction suppose that $\sqsubseteq^{II}_{\preceq}$ is $\preceq$-consistent. Thus, for the two assessments λ_V and $\lambda_{V'}$ such that $\lambda_V(x) = 1$ and $\lambda_{V'}(x) + \lambda_{V'}(y) = 1$ we have a utility function ν representing $\preceq$ such that

$$\nu(x)\lambda_V(x) > \nu(x)\lambda_{V'}(x) + \nu(y)\lambda_{V'}(y)$$

But $\nu(x) = \nu(y)$ because ν represents $\preceq$. Thus the precedent inequality gives us $\nu(x) > \nu(x)(\lambda_{V'}(x)+\lambda_{V'}(y))$, that is $\nu(x) > \nu(x)$, a contradiction.

A social choice correspondence satisfies *Citizens' Sovereignty* iff for any $x \in X$ there exists $u \in L^n$ such that $x \in f(u)$.

A social choice correspondence satisfies *Residual Resoluteness* iff for any $j \in N$; any pair $x, y \in X$; each $\preceq \in L$ such that the set of the two first element is $\{x, y\}$; each profile $u = (\preceq_1, \ldots, \preceq_n)$ in L^n such that for all $i \neq j$, $\preceq_i = \preceq$ and the two first elements of $\preceq_j$ are x and y and $\preceq_j\restriction_{X\setminus\{x,y\}} = \preceq\restriction_{X\setminus\{x,y\}}$, $f(u)$ is a singleton.

A social choice correspondence is *dictatorial* iff there exists i such that for all $u = (\preceq_1, \ldots, \preceq_n)$ in L^n, $f(u) = \text{mín}(X, \preceq_i)$.

Their result is the following theorem

Theorem 20. (Duggan-Schwartz [14]) *Suppose that X contains at least 3 alternatives. If a social choice correspondence f satisfies citizens' sovereignty and residual resoluteness then f is manipulable or f is dictatorial.*

Once stated this theorem, one can appreciate the great difference with our Theorem 16. In spite of the clearly different form, one may ask if our theorem is a corollary of Theorem 20. A first step in searching to answer this question is to know if we can drop out the condition that the social choice correspondences be defined only on linear orders. We can take a general social correspondence f, *i.e.* f is defined on all P^n and take the restriction, $\hat{f}$, to L^n. If $\hat{f}$ is manipulable, of course f is manipulable. The fact is that, doing this, a problem with the notion of dictator remains: one can easily define a general social correspondence f such that $\hat{f}$ is dictatorial but f is not. Another point that reveals why the approaches are different is that for any linear order $\preceq$, $\sqsubseteq^{II}_{\preceq}$ is not $\preceq$-consistent. To see that, it is enough to take $x \prec y \prec z$, $V = \{x, y\}$, $V' = \{x, z\}$, $\lambda_V(x) = \lambda'_V(x)$ and $\lambda_V(y) = \lambda'_V(z)$ and then use a technique similar to the one used above to prove that the lifting of a preference $\preceq'$ with ties is not $\preceq'$-consistent.

Concerning the work of Barberà et al. [2] and Benoit [5] we will make some comments. Both works consider a class of voting schemes but this time the type of objects considered has more structure. More formally, they consider some classes $\mathcal{D}$ of total pre-orders over agendas, and functions $f : \mathcal{D}^n \longrightarrow \mathcal{P}$. In the case of Barberà et al. the total pre-orders considered are of two classes. Both are defined in a quantitative way: the pre-orders considered are the pre-orders of expected utilities. The first class, denoted $\mathcal{D}_U$, contains the total pre-orders called *conditionally expected utility consistent*. It is defined in the following way: $\sqsubseteq \in \mathcal{D}_U$ iff there exists a utility function ν and a probability assessment over X, λ such that for any $V, V' \in \mathcal{P}^*(x)$

$$V \sqsubseteq V' \quad \Leftrightarrow \quad \sum_{x\in V} \nu(x)\left(\frac{\lambda(x)}{\lambda(V)}\right) \geq \sum_{x\in V'} \nu(x)\left(\frac{\lambda(x)}{\lambda(V')}\right)$$

The other class they consider is called *conditionally expected utility consistent with equal probabilities* and denoted $\mathcal{D}_E$. It is defined in the following way: $\sqsubseteq \in \mathcal{D}_E$ iff there exists a utility function ν such that for any $V, V' \in \mathcal{P}^*(x)$

$$V \sqsubseteq V' \quad \Leftrightarrow \quad \sum_{x\in V} \nu(x)\left(\frac{1}{|V|}\right) \geq \sum_{x\in V'} \nu(x)\left(\frac{1}{|V'|}\right)$$

Observation 21. *It is known that for any $\preceq$ preference over X, $\sqsubseteq^{II}_{\preceq}$ is in $\mathcal{D}_E$. The techniques to prove this are quite straightforward, one can see for instance [29, 30, 3, 4]. Nevertheless, this class is a strict subclass of $\mathcal{D}_E$,* i.e. *there are (many) elements in $\mathcal{D}_E$ which are different from $\sqsubseteq^{II}_{\preceq}$ for any $\preceq \in P$.*

A function $f : \mathcal{D}^n \longrightarrow \mathcal{P}^*(X)$ satisfies *unanimity* if for all profile $u = (\sqsubseteq_1, \ldots, \sqsubseteq_n)$ in which $\text{mín}(\mathcal{P}^*(X), \sqsubseteq_i) = V$ for all individual i, $f(u) = V$.

A function $f : \mathcal{D}^n \longrightarrow \mathcal{P}^*(X)$ is *dictatorial* if there exists an individual i such that for all profile $u = (\sqsubseteq_1, \ldots, \sqsubseteq_n)$ in $\mathcal{D}^n$, $f(u) \in \text{mín}(\mathcal{P}^*(X), \sqsubseteq_i)$.

A function $f : \mathcal{D}^n \longrightarrow \mathcal{P}^*(X)$ is *bi-dictatorial* if there exist individuals i, j such that for all profile $u = (\sqsubseteq_1, \ldots, \sqsubseteq_n)$ in $\mathcal{D}^n$, $f(u) \in \text{mín}(\mathcal{P}^*(X), \sqsubseteq_i) \cup \text{mín}(\mathcal{P}^*(X), \sqsubseteq_j)$.

Barberà et al. establish the following two theorems assuming that $|X| \geq 3$:

Theorem 22. (Barberà et al. [2]) *Let $f : \mathcal{D}_U^n \longrightarrow \mathcal{P}^*(X)$ such that f satisfies unanimity. Then f is manipulable or f is dictatorial*

Theorem 23. (Barberà et al. [2]) *Let $f : \mathcal{D}_E^n \longrightarrow \mathcal{P}^*(X)$ such that f satisfies unanimity. Then f is manipulable or f is dictatorial or bi-dictatorial.*

Again, once stated these theorems, one can appreciate the big difference with our Theorem 16. In fact, it is not possible to think about them in the same terms of Theorem 16. Actually when we try to "lift" a social choice correspondence $f : P^n \longrightarrow \mathcal{P}^*(X)$ to a function $\hat{f} : \mathcal{D}_U \longrightarrow \mathcal{P}^*(X)$ we face the problem that there are profiles $\hat{u} \in \mathcal{D}_U^n$ which are not the lifting of a profile $u \in P^n$. This happens because of observation 21.

In Benoit's work [5] there are also some classes of total pre-orders considered over $\mathcal{P}^*(X)$ and he establishes some theorems in the style of Theorem 22. The definition of Benoit's classes is quite technical. Nevertheless, it is a combinatorial one. What is interesting to observe is that the elements of these classes extend only linear orders over X. Due to this fact, we find essentially the same problems found with Duggan and Schwartz' work, when we try to generalize his results to all the class P^n. Furthermore, we find the problems commented above trying to lift social choice correspondences (even those defined on linear orders) to functions considered by Benoit.

A point that is worth mentioning is that the proofs of the results in the previous works are quite complex. On the contrary, our proofs are quite simple.

Summarizing our work is more general concerning the domains considered. The notion of weak dictator we consider is incomparable with the dictator notions of Duggan and Shwartz, Barberà et al. and Benoit because of the additional parameter, the agendas, that we consider. Our notion of manipulability is also quite different. Furthermore the manipulability result (Theorem 16) is very precise concerning the class of total pre-orders considered: there are two classes, the lifting type II and the lifting type IV.

6 Concluding remarks

Let us call the *two step approach to social choice* the way to calculate $f_u(V)$ in two steps: first calculate a sort of aggregation preference $\preceq_u$ and, second, calculate the minimal elements of V with respect to the preference $\preceq_u$. As we already saw, when f satisfies Transitive Explanations, f can be computed in such a way (cf. Proposition 6). Actually, given Proposition 6, the second step could seem superfluous because all information is encoded in $\preceq_u$. However the first remark after the results of previous Sections (in particular the weak result about manipulation, Theorem 16) is that the whole two step approach to Social Choice is fruitful and far from being superfluous. This new freedom degree -the agenda- together with the concept of lifting allow to define manipulability in a very natural way. Then, also in a very natural way, we can prove a manipulation theorem (we almost can say we *lift* the Gibbard-Satterthwaite's Theorem -Theorem 5).

Concerning this two step approach, it may be worth noting that it appears in Belief Merging under Integrity Constraints [18]. Actually, in that setting there are representation theorems very close to Proposition 6. Once more, we insist in the fact that what allows us to state these representation theorems is the explicit role of integrity constraints, the agendas in the framework of Social Choice, see [19].

Incorporating the agenda allowed us to give a different view of manipulability. Actually, a completely different one from those in which you need to have a ternary relation $\mathfrak{R}$ such that $\mathfrak{R}(\preceq, \preceq_1, \preceq_2)$ means that the preference $\preceq_1$ is closer to $\preceq$ than $\preceq_2$.

A by-product of this work has been to show clearly the big difference between voting schemes, social choice functions and social choice correspondences. In particular, there are many things to do concerning the manipulation in the framework of social choice functions. We list below some possible future work.

- To find more general manipulation theorems. As well with the agenda (the two step approach) as without it, *i.e.* with only the encoding $u \mapsto \preceq_u$ (one step approach). Some initial steps in this direction can be found in [20].
- To characterize the liftings for which Theorem 16 holds.
- To characterize the relations of closeness for which a manipulation theorem holds.

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