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Interior controllability of a 2×2 Reaction-Diffusion system with Cross Diffusion Matrix

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Resumen

En este artículo se presentan condiciones necesarias y suficientes para la controlabilidad exacta y aproximada del siguiente sistema diagonal por bloques, el cual representa una amplia clase de ecuaciones diferenciales de la forma:

$$z' = Az(t) + Bu(t), \quad t > 0, \quad z \in Z, \quad u \in U,$$

donde Z, U son espacios de Hilbert, $B \in L(U, Z)$, $u \in L^2(0, \tau; U)$ y $A : D(A) \subset Z \longrightarrow Z$ es el generador infinitesimal de un semigrupo $\{T(t)\}_{t \geq 0}$ en Z dado por:

$$T(t)z = \sum_{j=1}^{\infty} e^{A_j t} P_j z, \quad z \in Z, \quad t \geq 0$$

de acuerdo al Lema 2.1. Estos resultados serán aplicados a la ecuación de onda $n-$dimensional controlada, a la ecuación de calor de dimensión n y a la ecuación de una placa termoelástica.

Palabras clave: Sistema diagonal por bloques, controlabilidad, semigrupo fuertemente continuo, aplicaciones.

Abstract

In this paper we prove the interior approximate controllability for the following 2×2 Reaction-Diffusion System with Cross Diffusion Matrix

$$\begin{cases} u_t = a\Delta u - \beta(-\Delta)^{\frac{1}{2}} u + b\Delta v + 1_\omega f_1(t, x) & \text{in } (0, \tau) \times \Omega, \\ v_t = c\Delta u - d\Delta v - \beta(-\Delta)^{\frac{1}{2}} v + 1_\omega f_2(t, x) & \text{in } (0, \tau) \times \Omega, \\ u = v = 0, \quad \text{on } (0, T) \times \partial\Omega \\ u(0, x) = u_0(x), \quad v(0, x) = v_0(x), \quad x \in \Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$, $u_0, v_0 \in L^2(\Omega)$, the 2×2 Diffusion Matrix

$$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

has semi-simple and positive eigenvalues $0 < \rho_1 \leq \rho_2$, β is an arbitrary constant, ω is an open nonempty subset of Ω, 1_ω denotes the characteristic function of the set ω and the distributed controls $f_1, f_2 \in L^2([0, \tau]; L^2(\Omega))$. Specifically, we prove the following statement: If $\lambda_1^{\frac{1}{2}} \rho_1 + \beta > 0$(the first eigenvalue of $-\Delta$), then for all $\tau > 0$ and all open nonempty subset ω of Ω the system is approximately controllable on $[0, \tau]$.

key words. Interior controllability,reaction-diffusion system, strongly continuous semigroups.

AMS(MOS) subject classifications. Primary 93B05. Secondary 93C25.

1 Introduction.

In this paper we prove the interior approximate controllability for the following 2×2 Reaction-Diffusion System with Cross Diffusion Matrix

$$\begin{cases} u_t = a\Delta u - \beta(-\Delta)^{\frac{1}{2}}u + b\Delta v + 1_\omega f_1(t,x) & \text{in } (0,\tau)\times\Omega, \\ v_t = c\Delta u - d\Delta v - \beta(-\Delta)^{\frac{1}{2}}v + 1_\omega f_2(t,x) & \text{in } (0,\tau)\times\Omega, \\ u = v = 0, \quad \text{on } (0,\tau)\times\partial\Omega \\ u(0,x) = u_0(x), \quad v(0,x) = v_0(x), \quad x\in\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^N (N \geq 1)$, $u_0, v_0 \in L^2(\Omega)$, the 2×2 Diffusion Matrix

$$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{1.2}$$

has semi-simple and positive eigenvalues, β is an arbitrary constant, ω is an open nonempty subset of Ω, 1_ω denotes the characteristic function of the set ω and the distributed controls $f_1, f_2 \in L^2([0,\tau]; L^2(\Omega))$. Specifically, we prove the following statement: If $\lambda_1^{\frac{1}{2}}\rho_1 + \beta > 0$(the first eigenvalue of $-\Delta$), then for all $\tau > 0$ and all open nonempty subset ω of Ω the system is approximately controllable on $[0,\tau]$.

When $\Omega = (0,1)$ this system takes the following particular form

$$\begin{gathered} u_t = a\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial u}{\partial x} + b\frac{\partial^2 v}{\partial x^2} + 1_\omega f_1(t,x) \quad \text{in } (0,\tau)\times(0,1), \\ v_t = c\frac{\partial^2 u}{\partial x^2} + d\frac{\partial^2 v}{\partial x^2} + \beta\frac{\partial v}{\partial x} + 1_\omega f_2(t,x) \quad \text{in } (0,\tau)\times(0,1), \\ u(t,0) = v(t,0) = u(t,1) = v(t,1) = 0, \quad t\in(0,\tau), \\ u(0,x) = u_0(x), \quad v(0,x) = v_0(x), \quad x\in(0,1) \end{gathered} \tag{1.3}$$

This paper has been motivated by the work done S. Badraoui in [2], where author studies the asymptotic behavior of the solutions for the system (1.3) on the unbounded domain $\Omega = \mathbb{R}$. That is to say, he studies the system:

$$
\begin{aligned}
u_t &= a\frac{\partial^2 u}{\partial x^2} + \beta\frac{\partial u}{\partial x} + b\frac{\partial^2 v}{\partial x^2} + f(t,u,v), \quad x \in \mathbb{R},\ t > 0, \\
v_t &= c\frac{\partial^2 u}{\partial x^2} + d\frac{\partial^2 v}{\partial x^2} + \beta\frac{\partial v}{\partial x} + g(t,u,v), \quad x \in \mathbb{R},\ t > 0,
\end{aligned} \tag{1.4}
$$

supplemented with the initial conditions

$$
u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad x \in \mathbb{R}, \tag{1.5}
$$

where the diffusion coefficients a and d are assumed positive constants, while the diffusion coefficients b, c and the coefficient β are arbitrary constants. He assume also the following three conditions:

(H1) $(a-d)^2 + 4bc > 0$, $cd \neq 0$ and $ad > bc$.

(H2) $u_0, v_0 \in X = C_b(\mathbb{R})$.

(H3) $f(t,u,v)$ and $g(t,u,v) \in X$, for all $t > 0$ and $u, v \in X$. Moreover f and g are locally Lipshitz; namely, for all $t_1 \geq 0$ and all constant $k > 0$, there exist a constant $L = L(k, t_1) > 0$ such that

$$
|f(t,w_1) - f(t,w_2)| \leq L|w_1 - w_2|,
$$

is verified for all $w_1 = (u_1, v_1),\ w_2 = (u_2, v_2) \in \mathbb{R}\times\mathbb{R}$ with $|w_1| \leq k$, $|w_2| \leq k$ and $t \in [0, t_1]$.

We note that, the hypothesis H1) implies that the eigenvalues of the matrix D are simples and positive. But, this condition is not necessary for the eigenvalues of D to be positive, in fact we can find matrices D with a and d been negative and having positive eigenvalues. For example, one can consider the following matrix

$$
D = \begin{bmatrix} 5 & -6 \\ 2 & -2 \end{bmatrix},
$$

whose eigenvalues are $\rho_1 = 1$ and $\rho_2 = 2$.

The system (1.1) can be written in the following matrix form:

$$\begin{cases} z_t = D\Delta z - \beta I_{2\times 2}(-\Delta)^{\frac{1}{2}}z + 1_\omega f(t,x), & \text{in} \quad (0,\tau)\times\Omega, \\ z = 0, & \text{on} \quad (0,\tau)\times\partial\Omega, \\ z(0,x) = z_0(x), & x\in\Omega. \end{cases} \tag{1.6}$$

where $z = [u,v]^T \in \mathbb{R}^2$, the distributed controls $f = [f_1, f_2]^T \in L^2([0,\tau]; L^2(\Omega;\mathbb{R}^2))$ and $I_{2\times 2}$ is the identity matrix of dimension 2×2.

Our technique is simple and elegant from mathematical point of view, it rests on the shoulders of the following fundamental results:

Theorem 1.1 *The eigenfunctions of $-\Delta$ with Dirichlet Boundary Condition are real analytic functions.*

Theorem 1.2 *(see Theorem 1.23 from [1], pg. 20)*

Suppose $\Omega \subset I\!R^n$ is open, non-empty and connected set, and f is real analytic function in Ω with $f = 0$ on a non-empty open subset ω of Ω. Then, $f = 0$ in Ω.

Lemma 1.1 *(see Lemma 3.14 from [3], pg. 62)*

Let $\{\alpha_j\}_{j\geq 1}$ and $\{\beta_{i,j} : i = 1,2,\dots,m\}_{j\geq 1}$ be two sequences of real numbers such that: $\alpha_1 > \alpha_2 > \alpha_3 \cdots$. Then

$$\sum_{j=1}^{\infty} e^{\alpha_j t}\beta_{i,j} = 0, \quad \forall t\in[0,t_1], \quad i = 1,2,\cdots,m$$

iff

$$\beta_{i,j} = 0, \; i = 1,2,\cdots,m; j = 1,2,\cdots,\infty.$$

Finally, with this technique those young mathematicians who live in remote and inhospitable places, far from major research centers in the world, can also understand and enjoy the interior controllability with a minor effort.

2 Abstract Formulation of the Problem.

In this section we choose a Hilbert space where system (1.6) can be written as an abstract differential equation; to this end, we consider the following notations:

Let us consider the Hilbert space $H = L^2(\Omega, I\!R)$ and $0 = \lambda_1 < \lambda_2 < ... < \lambda_j \longrightarrow \infty$ the eigenvalues of $-\Delta$, each one with finite multiplicity γ_j equal to the dimension of the corresponding eigenspace. Then, we have the following well known properties:

(i) There exists a complete orthonormal set $\{\phi_{j,k}\}$ of eigenvectors of $-\Delta$.

(ii) For all $\xi \in D(-\Delta)$ we have

$$-\Delta\xi = \sum_{j=1}^{\infty} \lambda_j \sum_{k=1}^{\gamma_j} < \xi, \phi_{j,k} > \phi_{j,k} = \sum_{j=1}^{\infty} \lambda_j E_j \xi, \tag{2.7}$$

where $< \cdot, \cdot >$ is the inner product in H and

$$E_n x = \sum_{k=1}^{\gamma_j} < \xi, \phi_{j,k} > \phi_{j,k}. \tag{2.8}$$

So, $\{E_j\}$ is a family of complete orthogonal projections in H and
$\xi = \sum_{j=1}^{\infty} E_j \xi, \quad \xi \in H.$

(iii) Δ generates an analytic semigroup $\{T_\Delta(t)\}$ given by

$$T_\Delta(t)\xi = \sum_{j=1}^{\infty} e^{-\lambda_j t} E_j \xi. \tag{2.9}$$

Now, we denote by Z the Hilbert space $H^2 = L^2(\Omega; \mathbb{R}^2)$ and define the following operator

$$A : D(A) \subset Z \longrightarrow Z, \quad A\psi = -D\Delta\psi + \beta I_{2\times 2}(-\Delta)^{\frac{1}{2}}\psi$$

with $D(A) = H^2(\Omega, I\!R^2) \cap H_0^1(\Omega, I\!R^2)$.

Therefore, for all $z \in D(A)$ we obtain,

$$Az = \sum_{j=1}^{\infty} \lambda_j^{\frac{1}{2}} (\lambda_j^{\frac{1}{2}} D + \beta I_{2\times 2}) P_j z, \tag{2.10}$$

and

$$z = \sum_{j=1}^{\infty} P_j z, \quad \| z \|^2 = \sum_{j=1}^{\infty} \| P_j z \|^2, \quad z \in Z,$$

where

$$P_j = \begin{bmatrix} E_j & 0 \\ 0 & E_j \end{bmatrix}$$

is a family of complete orthogonal projections in Z.

Consequently, system (1.6) can be written as an abstract differential equation in Z:

$$z' = -Az + B_\omega f, \quad z \in Z \quad t \geq 0, \tag{2.11}$$

where $f \in L^2([0,\tau];U)$, $U = Z$ and $B_\omega : U \longrightarrow Z$, $B_\omega f = 1_\omega f$ is a bounded linear operator.

Now, we shall use the following Lemma from [5] to prove the next Theorem.

LEMMA 2.1 *Let Z be a Hilbert separable space and $\{A_j\}_{j\geq 1}$, $\{P_j\}_{j\geq 1}$ two families of bounded linear operator in Z, with $\{P_j\}_{j\geq 1}$ a family of complete orthogonal projection such that:*

$$A_j P_j = P_j A_j, j \geq 1.$$

Define the following family of linear operators

$$T(t)z = \sum_{j=1}^{\infty} e^{A_j t} P_j z, \;\; z \in Z, \;\; t \geq 0.$$

Then:

(a) $T(t)$ is a linear and bounded operator if $\|e^{A_j t}\| \leq g(t)$, $j = 1, 2, ...,$ with $g(t) \geq 0$, continuous for $t \geq 0$.

(b) Under the above (a), $\{T(t)\}_{t\geq 0}$ is a strongly continuous semigroup in the Hilbert space Z, whose infinitesimal generator A is given by

$$Az = \sum_{j=1}^{\infty} A_j P_j z, \quad z \in D(A)$$

with

$$D(A) = \left\{ z \in Z : \sum_{j=1}^{\infty} \|A_j P_j z\|^2 < \infty \right\}.$$

(c) The spectrum $\sigma(A)$ of A is given by

$$\sigma(A) = \overline{\bigcup_{j=1}^{\infty} \sigma(\overline{A_j})},$$

where $\overline{A_j} = A_j P_j : \mathcal{R}(P_j) \to \mathcal{R}(P_j)$.

THEOREM 2.1 *The operator $-A$ define by (2.10), is the infinitesimal generator of a strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ given by:*

$$T(t)z = \sum_{j=1}^{\infty} e^{A_j t} P_j z, \quad z \in Z, \quad t \geq 0, \tag{2.12}$$

where $P_j = diag[E_j, E_j]$ and $A_j = R_j P_j$ with

$$R_j = \begin{bmatrix} -a\lambda_j & -b\lambda_j \\ -c\lambda_j & -d\lambda_j \end{bmatrix} - \beta \begin{bmatrix} \lambda_j^{\frac{1}{2}} & 0 \\ 0 & \lambda_j^{\frac{1}{2}} \end{bmatrix}.$$

Moreover, if $\lambda_1^{\frac{1}{2}} \rho_1 + \beta > 0$, then there exists $M > 0$ such that

$$\|T(t)\| \leq M \exp\{-\lambda_1^{\frac{1}{2}} (\lambda_1^{\frac{1}{2}} \rho_1 + \beta) t\}, \quad t \geq 0. \tag{2.13}$$

Proof . In order to apply the foregoing Lemma, we observe that $-A$ can be written as follows:

$$Az = \sum_{j=1}^{\infty} A_j P_j z, \quad z \in D(A)$$

with

$$A_j = -\lambda_j^{\frac{1}{2}} (\lambda_j^{\frac{1}{2}} D + \beta I_{2\times 2}) P_j \ \text{ and } \ P_j = \text{diag}[E_j, E_j].$$

Therefore, $A_j = R_j P_j$ with

$$R_j = \begin{bmatrix} -a\lambda_j & -b\lambda_j \\ -c\lambda_j & -d\lambda_j \end{bmatrix} - \beta \begin{bmatrix} \lambda_j^{\frac{1}{2}} & 0 \\ 0 & \lambda_j^{\frac{1}{2}} \end{bmatrix} \quad \text{and} \quad A_j P_j = P_j A_j.$$

Now, we have to verify condition (a) of Lemma 2.1. To this end, without lose of generality, we shall suppose that $0 < \rho_1 < \rho_2$. Then, there exists a set $\{Q_1, Q_2\}$ of complementary projections on $\mathbb{R}^2$ such that:

$$e^{Dt} = e^{\rho_1 t} Q_1 + e^{\rho_2 t} Q_2.$$

Hence,

$$e^{R_j t} = e^{\Gamma_{1j} t} Q_1 + e^{\Gamma_{2j} t} Q_2, \quad \text{with} \quad \Gamma_{js} = -\lambda_j^{\frac{1}{2}} \left[\lambda_j^{\frac{1}{2}} \rho_s + \beta \right], \quad s = 1, 2.$$

This implies the existence of positive numbers α, M such that

$$\|e^{A_j t}\| \leq M e^{\alpha t}, j = 1, 2, ...$$

Therefore, $-A$ generates a strongly continuous semigroup $\{T(t)\}_{t \geq 0}$ given by (2.12).

Finally, if $\lambda_1^{\frac{1}{2}} \rho_1 + \beta > 0$, then

$$-\lambda_1^{\frac{1}{2}} (\lambda_1^{\frac{1}{2}} \rho_1 + \beta) \geq -\lambda_j^{\frac{1}{2}} (\lambda_j^{\frac{1}{2}} \rho_i + \beta), \quad j = 1, 2, 3, \ldots; i = 1, 2,$$

and using (2.12) we obtain (2.13). □

3 Proof of the Main Theorem

In this section we shall prove the main result of this paper on the controllability of the linear system (2.11). But, before we shall give the definition of approximate controllability for this system. To this end, for all $z_0 \in Z$ and $f \in L^2(0, \tau; U)$ the initial value problem

$$\begin{cases} z' = -Az + B_\omega f(t), z \in Z \\ z(0) = z_0 \end{cases} \tag{3.14}$$

where the control function f belong to $L^2(0,\tau;U)$ admits only one mild solution given by

$$z(t) = T(t)z_0 + \int_0^t T(t-s)B_\omega f(s)ds, \ \ t \in [0,\tau]. \tag{3.15}$$

Definition 3.1 *(**Approximate Controllability**) The system (2.11) is said to be approximately controllable on $[0,\tau]$ if for every z_0, $z_1 \in Z$, $\varepsilon > 0$ there exists $u \in L^2(0,\tau;U)$ such that the solution $z(t)$ of (3.15) corresponding to u verifies:*

$$\|z(\tau) - z_1\| < \varepsilon.$$

The following result can be found in [4] for the general evolution equation

$$z' = \mathcal{A}z + Bf(t), \quad z \in Z, \quad u \in U, \tag{3.16}$$

where Z, U are Hilbert spaces, $\mathcal{A} : D(\mathcal{A}) \subset Z \longrightarrow Z$ is the infinitesimal generator of strongly continuous semigroup $\{T(t)\}_{t\geq 0}$ in $Z, B \in L(U,Z)$, the control function f belong to $L^2(0,\tau;U)$.

Theorem 3.1 *The system (3.16) is approximately controllable on $[0,\tau]$ if, and only if,*

$$B^*T^*(t)z = 0, \quad \forall t \in [0,\tau] \Rightarrow z = 0. \tag{3.17}$$

Now, we are ready to formulate and prove the main theorem of this work.

Theorem 3.2 *If $\lambda_1^{\frac{1}{2}}\rho_1 + \beta > 0$, then for all $\tau > 0$ and all open nonempty subset ω of Ω the system (2.11) is approximately controllable on $[0,\tau]$.*

Proof . We shall apply Theorem 3.1 to prove the approximate controllability of system (2.11). With this purpose, we observe that

$$B_\omega^* = B_\omega \text{ and } T^*(t)z = \sum_{j=1}^{\infty} e^{R_j^* t} P_j^* z, \quad z \in Z, \quad t \geq 0.$$

On the other hand,

$$R_j = -\lambda_j^{\frac{1}{2}} \left\{ \lambda_j^{\frac{1}{2}} \begin{bmatrix} a & b \\ c & d \end{bmatrix} + \beta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\} = -\lambda_j^{\frac{1}{2}} \left\{ \lambda_j^{\frac{1}{2}} D + \beta I_{2\times 2} \right\}.$$

Without lose of generality, we shall suppose that $0 < \rho_1 < \rho_2$. Then, there exists a set $\{Q_1, Q_2\}$ of complementary projections on $\mathbb{R}^2$ such that:

$$e^{Dt} = e^{\rho_1 t}Q_1 + e^{\rho_2 t}Q_2.$$

Hence,

$$e^{R_j t} = e^{\Gamma_{1j} t} Q_1 + e^{\Gamma_{2j} t} Q_2, \quad \text{with} \quad \Gamma_{js} = -\lambda_j^{\frac{1}{2}} \left[\lambda_j^{\frac{1}{2}} \rho_s + \beta \right], \quad s = 1, 2.$$

Therefore,

$$B_\omega^* T^*(t) z = \sum_{j=1}^{\infty} B_\omega^* e^{R_j * t} P_j^* z = \sum_{j=1}^{\infty} \sum_{s=1}^{2} e^{\Gamma_{sj} t} B_\omega^* P_{s,j}^* z,$$

where $P_{s,j} = Q_s P_j = P_j Q_s$.

Now, suppose for $z \in Z$ that $B_\omega^* T^*(t) z = 0, \quad \forall t \in [0, \tau]$. Then,

$$\begin{aligned} B_\omega^* T^*(t) z \quad &= \quad \sum_{j=1}^{\infty} B_\omega^* e^{R_j^* t} P_j^* z = \sum_{j=1}^{\infty} \sum_{s=1}^{2} e^{\Gamma_{sj} t} B_\omega^* P_{s,j}^* z = 0. \\ &\Longleftrightarrow \quad \sum_{j=1}^{\infty} \sum_{s=1}^{2} e^{\Gamma_{sj} t} (B_\omega^* P_{s,j}^*) z(x) = 0, \quad \forall x \in \Omega. \end{aligned}$$

Clearly that, $\{\Gamma_{sj}\}$ is a decreasing sequence. Then, from Lemma 1.1, we obtain for all x $\in \Omega$ that

$$(B_\omega^* P_{s,j}^* z)(x) = Q_s^* \begin{bmatrix} \sum_{k=1}^{\gamma_j} < z_1, \phi_{j,k} > 1_\omega \phi_{j,k}(x) \\ \sum_{k=1}^{\gamma_j} < z_2, \phi_{j,k} > 1_\omega \phi_{j,k}(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, j = 1, 2, 3, 4, \ldots; s = 1, 2.$$

Since $Q_1 + Q_2 = I_{\mathbb{R}^2}$, we get that

$$\begin{bmatrix} \sum_{k=1}^{\gamma_j} < z_1, \phi_{j,k} > \phi_{j,k}(x) \\ \sum_{k=1}^{\gamma_j} < z_2, \phi_{j,k} > \phi_{j,k}(x)) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, j = 1, 2, 3, 4, \ldots; s = 1, 2, \quad \forall x \in \omega.$$

On the other hand, from Theorem 1.1 we know that $\phi_{n,k}$ are analytic functions, which implies the analyticity of $E_j z_i = \sum_{k=1}^{\gamma_j} < z_i, \phi_{j,k} > \phi_{j,k}$. Then, from Theorem 1.2 we get that

$$\begin{bmatrix} \sum_{k=1}^{\gamma_j} < z_1, \phi_{j,k} > \phi_{j,k}(x) \\ \sum_{k=1}^{\gamma_j} < z_2, \phi_{j,k} > \phi_{j,k}(x) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, j = 1, 2, 3, 4, \ldots, \quad \forall x \in \Omega, \quad s = 1, 2.$$

Hence $P_j z = 0, j = 1, 2, 3, 4, \ldots$, which implies that $z = 0$. This completes the proof of the main Theorem. □

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